# Coordinate systems: Polar and Rectangular Bearing: Magnetic and True, Whole-circle and Reduced 

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## Co-ordinate systems: Polar and Rectangular

In Surveying, one of the primary functions is to describe or establish the positions of points on the surface of the earth. One of the many ways to accomplish this is by using coordinates to provide an address for the point.

Co-ordinates are used to locate a position. Two such systems of co-ordinates are:
(i) Polar and (ii) Rectangular co-ordinates

## Polar Coordinates:

The coordinate system which depicts the points from the pole to the certain distance at a particular angle is known as 'polar coordinates'. The positive horizontal axis in which that particular angle rotates is named as 'polar axis.


Assume line OP of length, $r$ making an angle $\theta$ with any reference line OX . The Polar coordinates of P are $r, \theta$. Drop a perpendicular PM . If OM and PM are required, the values are: $\mathrm{OM}=r \cos \theta$ and $\mathrm{PM}=r \sin \theta$
** $\cos \theta=\frac{O M}{r}$
$\therefore \mathrm{OM}=r \cos \theta$
$\sin \theta=\frac{P M}{r}$
$\therefore \mathrm{PM}=r \sin \theta$

## Representation of axes system for polar coordinates:

Consider the centre as origin $(0,0)$ and radius $r$. Rotate the point in a circular path by keeping the radius $r$ as constant. The illustration of $60^{\circ}$ is as follows:


## Rectangular coordinates:

The rectangular coordinate indicates the position of points with respect to fixed point as reference and is the shortest distance from axes. The rectangular coordinates are also known as Cartesian coordinates. In two dimensional coordinate system, the fixed axes are the x and y axes. For any coordinate system, the axes are perpendicular to each other and met at the fixed point is known as the origin. In other words, the rectangular coordinates represent the location of a point which is at a perpendicular distance from two lines.

In a two dimensional system, the horizontal axis is known as x -axis and the vertical axis is known y -axis. A rectangular coordinate is usually represented as an ordered pair ( $\mathrm{x}, \mathrm{y}$ ). The value x is horizontal distance from the origin and y is the vertical distance from the origin. Moreover, the x coordinate is known as abscissa and y coordinate is known as ordinate.


The location of a point $P$ is known by $x$-distance, $O M$ and $y$-distance, $P M$ i.e. by $x$-coordinate and $y$ coordinate respectively. If the distance OP is required, it can be calculated by,
$\left.\sqrt{\left(O M^{2}\right.}+O P^{2}\right)$ i.e. by $\left.\sqrt{\left(x^{2}\right.}+y^{2}\right)$

## Conversion of Polar co-ordinates to Rectangular co-ordinates and vice versa.

- Polar to Rectangular co-ordinates:

$$
\text { Polar } \left.\left\{\begin{array}{lll}
r & =50 \mathrm{~m} \\
\theta & =10^{\circ}
\end{array}\right] \begin{array}{rlll}
\mathrm{x} & =r \cos \theta & =50 \cos 10^{\circ} & =49.24 \mathrm{~m} \\
\mathrm{y} & =r \sin \theta & =50 \sin 10^{\circ} & =8.68 \mathrm{~m}
\end{array}\right\} .
$$

- Rectangular to Polar co-ordinates:

Rectangular $\left\{\begin{array}{l}x=49.24 m \\ y=8.68 m\end{array}\right.$

$$
\left.\theta=\tan ^{-1} \frac{y}{x}=\tan ^{-1} \frac{8.68}{49.24}=10^{\circ} \quad\right]_{\text {Polar }}
$$

$$
\left.r=\sqrt{\left(x^{2}\right.}+y^{2}\right)=\sqrt{\left((49.24)^{2}+(8.68)^{2}\right)} \quad=50 \mathrm{~m}
$$

## Bearing: Magnetic and True, Whole-circle and Reduced

Bearing of lines: The bearing of a line is the horizontal angle which the line makes with some reference direction or meridian. The reference direction employed in surveying may be
(i) a true meridian,
(ii) a magnetic meridian, or
(iii) an arbitrary or assumed-meridian.

The true meridian is usually employed in geodetic surveys, while the magnetic meridian is used in plane surveys.

True Meridians: The points of intersection of the earth's axis and the surface of the earth are known as the north geographical pole and the south geographical pole. The true or geographical meridian passing through a point on the earth's surface is the line in which the plane passing through the given point and the north and south poles intersects the surface of the earth. The direction of a true meridian at a station is invariable. The true meridians through the various stations are not parallel, but converge to the poles. However, for ordinary small surveys, they are assumed to be parallel, to each other. The horizontal angle between the true meridian and a line is called a true bearing of the line. It is also known as an azimuth.

Magnetic Meridian: The direction indicated by a freely suspended and properly balanced magnetic needle, unaffected by local attractive forces is called the magnetic meridian or the magnetic north and south line. The angle which a line makes with the magnetic meridian is called a magnetic bearing of the line or simply a bearing of the line.

Arbitrary Meridian: For small surveys any convenient direction may be taken as a meridian. It is usually the direction from a survey station to some well-defined permanent object, or the first line of a survey. The angle between this meridian and a line is known as an arbitrary or assumed bearing of the line.

Designation of Bearings; There are two systems of notation commonly used to express bearings viz. (1) the whole circle system and (2) the quadrantal system.

Whole Circle System: In this system the bearing of a line is always measured clockwise from the north point of the reference meridian towards the line right round the circle. The angle thus measured is called the whole circle bearing (W C.B). It may have any value between $0^{\circ}$ to $360^{\circ}$. Thus in Fig. 1 the W.C.B. of $A P_{1}$ is $\theta_{1}$, that of $\mathrm{AP}_{2}$ is $\theta_{2}$; and so on.

In this system the bearing is completely specified by the angle and the noting of the cardinal points $\mathrm{N}, \mathrm{E}$, S and W is not required.

The bearings observed with a prismatic compass or a theodolite is the whole circle bearings.
Quadrantal System: In this system the bearing of a line is measured clockwise or counterclockwise from the north point or the south point whichever - is nearer the line, towards the east or west. It is, therefore,
absolutely necessary to state the point from which the angle is measured and also the direction in which it is measured.

The plane around a station is divided into four quadrants by the two lines at right angles to each other, of which one is the north and south line and the other the east and west line. The letters N (north), S (south), E (east) and W (west) are used to show the quadrants. Thus in Fig. 2 the first quadrant is denoted by the letters N.E., the second one by the letters S.E., the third one by the letters S.W., and the fourth one by the letters N.W.

In this system the bearing is reckoned from $0^{\circ}$ to $90^{\circ}$ each quadrant. The quadrantal bearing, therefore, never exceeds $90^{\circ}$. There are two notations in which the bearing of line is expressed. In the first notation the letters showing the quadrant in which the line falls are put after the numerical value of the angle. Thus the bearing of $\mathrm{AP}_{1}$ is $\theta_{1}$ N.E.; that of $\mathrm{AP}_{2}, \theta_{2}$ S.E.; and so on.


Fig. 1
Fig. 2
In the second notation which is more commonly used, the numerical value of the bearing is preceded by the letter N . or S . and followed by the letter E . or W . Thus the bearing of $\mathrm{AP}_{3}$ is $\mathrm{S} \theta_{3} \mathrm{~W}$; that of $\mathrm{AP}_{4} \mathrm{~N} \theta_{4}$ W ; and so on. It must he remembered that the quadrantal bearings are never reckoned from the east and west line. They are often called the reduced hearings.

The quadrantal system is an advantage when finding the values of the trigonometrical functions from the logarithmic tables. But the disadvantages of the system are: (i) the bearing is of no value, if either of the letters showing the quandrant is omitted, and (ii) the noting of the cardinal points is inconvenient, and may cause mistakes. The bearings observed with a surveyor's compass are the quadrantal bearings.

Reduced Bearings: When the whole circle bearing of a line exceeds $90^{\circ}$, it must be reduced to the corresponding angle less than $90^{\circ}$ which has the same numerical values of the trigonometrical functions. This angle is known as the reduced bearing (R.B.). To obtain the reduced bearings from the whole circle bearings of lines (Fig. 3), the following table may be referred to:

| Case | W.C.B. between | Rule for R.B. | Quadrant |
| :---: | :---: | :--- | :---: |
| I | $0^{\circ}$ and $90^{\circ}$ | $=$ W.C. B. | N.E. |
| II | $90^{\circ}$ and $180^{\circ}$ | $=180^{\circ}-$ W.C. B. | S.E. |
| III | $180^{\circ}$ and $270^{\circ}$ | $=$ W.C. B. $-180^{\circ}$ | S.W. |
| IV | $270^{\circ}$ and $360^{\circ}$ | $=360^{\circ}-$ W.C. B. | N.W. |



Fig. 3
Fore and Back Bearings: Every line has two bearings one observed at each end of the line. The bearing of a line in the direction of the progress of survey is called the fore or forward bearing (F.B.) while its bearing in the opposite direction is known as the back or reverse bearing (B.B.). The end of the line at which the bearing is taken is indicated by the order in which the line is lettered (i.e. given first). Thus in Fig. 4 the bearing from $A$ to $B$ is the forebearing of the line $A B$, and that from $B$ to $A$ is the back bearing of the line AB or the bearing of the BA . It will be noticed here that the fore and back bearings of a line differ exactly by $180^{\circ}$. In the whole circle bearing system the back bearing of a line may be obtained from the fore bearing by the following rule:

## Back bearing $=$ Fore bearing $\pm 180^{\circ}$

Use plus sign, if the given fore bearing is less than $180^{\circ}$ and minus sign, if it exceeds $180^{\circ}$.
In the quadrantal system the fore and back bearings are numerically equal but with opposite letters. The back bearing of a line may, therefore, be obtained by simply substituting N. for S. or S. for N., and E. for W . or W. for E . Thus, if the fore bearing of a line CD is $\mathrm{N} 40^{\circ} 25^{\prime} \mathrm{E}$, the back, bearing of CD is $\mathrm{S} 40^{\circ} 25^{\prime}$ W.


Magnetic Declination: (Fig.5) The magnetic meridian (i.e. the horizontal direction adopted by the geometrical axis of the needle unaffected by local attraction) at a place does not coincide with the true meridian at that place except in few places. The horizontal angle which the magnetic meridian makes with the true or geographical meridian is known as the magnetic declination or the declination of the needle. When the north end of the needle points to the east of the true meridian, the declination is said to be east $\left(n^{\circ} E\right)$; when the north end of the needle points to the west of the true meridian, the declination is said to be west ( $n^{\circ}$ W.). In some places the needle is deflected east of the true north and in others it points to the west of the true north. Since the magnetic meridian varies from place to place on the earth's surface, the amount and direction of the declination is different in different localities. If the true bearing of a line is determined by astronomical observations, the declination at any place can be found by observing the magnetic bearing of that line and finding their difference, or can be obtained approximately from isogonic charts published from time to time.


