Example 17 Prove that $\log \frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$

Solution: L.H.S $= \log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243}$

$= \log 75 - \log 16 - 2\log 5 + 2\log 9 + \log 32 - \log 243$

$= \log(3\times 5^2) - \log 2^4 - 2\log 5 + 2\log 3^2 + \log 2^5 - \log 3^5$

$= \log 3 + 2\log 5 - 4\log 2 - 2\log 5 + 4\log 3 + 5\log 2 - 5\log 3$

$= \log 2 \quad = \text{R.H.S}$

Example 18 If $\log_{10} 2 = 0.30103$, what is the value of $\log_{10} 2000$?

Solution: We are given that $\log_{10} 2 = 0.30103$

Now, $\log_{10} 2000 = \log_{10}(2\times 1000)$

$= \log_{10} 2 + \log_{10} 1000$

$= \log_{10} 2 + \log_{10} 10^3$

$= \log_{10} 2 + 3\log_{10} 10$

$= \log_{10} 2 + 3 \quad (\because \log_{10} 10 = 1)$

$= 0.30103 + 3$

$= 3.30103$

Exercises: 1. If $\log_p x = a$, $\log_q x = b$, then prove that $\log_{pq} x = \frac{ab}{a+b}$

2. Show that $23\log_{10}\frac{16}{15} + 17\log_{10}\frac{25}{24} + 10\log_{10}\frac{81}{80} = 1$

3. Show that $\dfrac{\log\sqrt{27} + \log\sqrt{8} - \log\sqrt{125}}{\log 6 - \log 5} = \dfrac{3}{2}$

4. If $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y}$, then prove that $xyz = 1$

5. Prove that $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$

6. If $\dfrac{\log x}{b+c} = \dfrac{\log y}{c+a} = \dfrac{\log z}{a+b}$, where $x>0$, $y>0$, and $z>0$,

then prove that $x^b y^c z^a = x^c y^a z^b$

7. Prove that $(yz)^{\log y - \log z} \times (zx)^{\log z - \log x} \times (xy)^{\log x - \log y} = 1$

8. If $\log_2 x + \log_4 x + \log_{10} x = \frac{21}{4}$, find x,

9. Show that the value of $\log_{10} 2$ lies between $\frac{1}{3}$ and $\frac{1}{4}$

10. If $\log(x^2y^3) = a$ and $\log\left(\frac{x}{y}\right) = b$, find $\log x$ and $\log y$ in terms of a and b.

11. Prove that $\dfrac{1}{\log_{ab}(abc)} + \dfrac{1}{\log_{bc}(abc)} + \dfrac{1}{\log_{ca}(abc)} = 2$

12. If $x^2 + y^2 = 6xy$, prove that $2\log(x+y) = \log x + \log y + 3\log 2$

13. Prove that $\log(1+2+3) = \log 1 + \log 2 + \log 3$

14. If $\frac{a(y-z)}{\log a} = \frac{b(z-x)}{\log b} = \frac{c(x-y)}{\log c}$, then prove that $a^{x/a}\, b^{y/b}\, c^{z/c} = 1$

15. If $y = a^{\frac{1}{1-\log_a x}}$, $z = a^{\frac{1}{1-\log_a y}}$, then prove that $x = a^{\frac{1}{1-\log_a z}}$

16. If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, prove that

$$xyz = x+y+z+2$$

17. Solve: $\log_x 2 \cdot \log_{\frac{x}{16}} 2 = \log_{\frac{x}{64}} 2$

18. If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$, find x.

(Earlier chapter written wrongly as 5, so, we are writing here this chapter as 4)

4. SET THEORY

4.1 Set Any well defined collection of distinct objects. The objects may be anything: numbers, people, books, letters of the alphabet, rivers, lines and points in geometry, etc. Each object of the set is called its element or member. Sets are usually denoted by capital letters like A, B, M, N etc and the elements by small letters such as a, b, c, m, n, and t etc

Examples of sets:

(a) The set of numbers 2, 4, 7 and 8

(b) The set of vowels in the English alphabet

(c) The set of positive integers that are multiples of 4

(d) The set of students of B.Com. Sem-III of Calcutta University.

e) The set of rivers in West Bengal

f) The set of real numbers lying between 0 and 1

g) The set of integers.

4.2 METHODS OF SET REPRESENTATION AND NOTATION

There are two methods of representing a set – either by showing a list of its elements (called roster method) or by stating some properties that decide whether a particular object is an element of the set or not (called property method).

Let us suppose that 1, 3, 5, 7, 9 are the elements of a set A. Then by roster method, we write

$$A = \{1, 3, 5, 7, 9\}$$

That is, the elements of the set are written down one after another separated by commas and then enclosed in a pair of curly brackets or braces. It may be mentioned that the arrangement of elements in the set is immaterial. We may also write the set A as

$$A = \{5, 7, 1, 3, 9\} = \{5, 1, 3, 7, 9\} \text{ etc.}$$

Again, by the property method, we may write the above set by stating characteristic property of its elements, in the form

$$A = \{x : x \text{ is a positive odd integer less than } 10\}$$