This is read " A is the set of elements x, such that x is a possitive odd integer less than 10".

Sometimes also we write $A = \{x \mid x$ is a positive integer less than $10\}$

Note: The symbol ':' or '|' is used to denote "such that".

So, the sets listed in section 4.1 may be written as follows :

(a) $\{2, 4, 7, 8\}$

(b) $\{a, e, i, o, u\}$

(c) ~~{Head, Tail}~~ $\{4, 8, 12, 16, \cdots\}$

~~(d) {1, 2, 3, 4, 5, 6}~~

(d) $\{x : x$ is a student of B.Com. ~~Semester~~ Semester-III of Calcutta University$\}$

(e) $\{x : x$ is a river in West Bengal$\}$

(f) $\{x \mid x$ is a real number, $0 \leq x \leq 1\}$

(g) $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

4.2.1 Notation : Element of a set

If a is an element of a set A, we write

$$a \in A$$

This is read as "a belongs to A". If b is not an element of A, then we write $b \notin A$ and say b does not belong to A.

Examples (a) If $A = \{2, 4, 7, 9\}$ then $2 \in A$, $3 \notin A$

(b) If $B = \{x : x^2 - 5x + 6 = 0\}$ then $3 \in B$, $8 \notin B$

as 2 and 3 are the only elements of B.

4.3.0 Types of sets

Different types of sets are discussed Below.

4.3.1 Null set : A set that contains no element is called a null set (also known as empty set or void set) It is denoted by the symbol ϕ. The concept of such a set is necessary in the set theory

The following sets are all null sets:

(a) $\{x : x \text{ is a multiple of } 4 \text{ and } x \text{ is odd}\}$

(b) $\{x : x^2+4=0, x \text{ is real}\}$

4.3.2 Singleton set

A set that has only one element is called a singleton set.

Examples : (a) $A = \{3\}$ is a singleton set

(b) $B = \{0\}$ is a singleton set.

(c) The set $\{x : 3x+4=0\}$ is a singleton set.

4.3.3 Finite and Infinite set

A set is said to be finite, if it is empty or contains a specific number of elements (i.e., if the number of elements can be counted and the counting process can be completed)

A non-empty that does not contain a specific number of elements is called an infinite set

Infinite sets are of two types : (i) Denumerable and (ii) Non-denumerable. An infinite set is called denumerable, if its element can be arranged in the form of a sequence.

An infinite set whose elements can not be given in the form of a sequence is called non-denumerable set.

A set which is finite or denumerable is called a Countable set. A non-denumerable set is also called an uncountable set.

Examples (a) The set $\{1, 3, 4, 7, 9\}$ is a finite set because the number of elements of the set can be stated by a specific number 5.

(b) The set of days of a week is a finite set

(c) The set of positive even integer $\{2, 4, 6, \cdots\}$ is an infinite set, because the total number of elements of the set can not be stated by a specified number. It is also a denumerable set, because its elements can be written down in the increasing order by a specific rule. The 100th element of the set, for example, is known to be 200. i.e., nth element of the set can be written as $2n$.

(d) The set $\{x : x \text{ is real}, 0 \leq x \leq 1\}$ is an infinite and non-denumerable set because the number of elements of the set can neither be stated by any specific figure nor the elements can be arranged in a sequence.

(e) Let $M = \{x : x \text{ is a mountain peak in the world}\}$. Although it is difficult to count all the mountain peaks in the whole world, M is still a finite set

4.3.4 Equal sets.

Two sets A and B are said to be equal if they have the same elements. and we write $A = B$

This means That every element of A is also an element of B, and every element of B is also an element of A.

When two sets A & B are not equal we write $A \neq B$.

(a) $A = \{3, 7, 8\}$ and $B = \{8, 7, 3\}$. Then $A = B$

(b) $C = \{2, 3\}$ $D = \{x : x^2 - 5x + 6 = 0\}$ and
$E = \{3, 2, 2, 3\}$ Then $C = D = E$

(c) The sets $P = \{a, b, c, d\}$ and $Q = \{a, c, d, e\}$ are not equal i.e., $P \neq Q$.

(d) The set $A = \{x : x^2 + 4 = 0, x \text{ is real}\}$ and $B = \{x : x^2 + 9 = 0, x \text{ is real}\}$ are equal because A and B are null sets. So, $A = B = \phi$

4.3.5 Subset

If every element of a set A is also an element of B, then A is said to be a subset of B, and we write $A \subseteq B$ or $B \supseteq A$

Symbolically, we write $A \subseteq B$, if $x \in A \Rightarrow x \in B$

The sign '$\Rightarrow$' used to denote implies that

If A is not a subset of B, we write $A \nsubseteq B$

The following results are important:

(1) Every set is a subset of itself, i.e., $A \subseteq A$