(ii) The null set ϕ is considered as a subset of any set A, i.e., $\phi \subseteq A$

(iii) If A is a subset of B, and B is a subset of C, then A is a subset of C. In symbol, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$

Proper subset Any set A is said to be a proper subset of another set B, if A is a subset of B, but there is at least one element of B that does not belong to A. We denote this by writing $A \subset B$.

It may be noted, $A \subseteq B$, if $x \in A \Rightarrow x \in B$

$A \subset B$, if $x \in A \Rightarrow x \in B$ and $A \neq B$

Examples (a) The set $A = \{1, 3, 4\}$ is a subset of $B = \{1, 2, 3, 4, 6\}$. Here $A \subset B$

(b) Let $C = \{3, 7, 8\}$ $D = \{8, 3, 7\}$

Then $C \subseteq D$ and $D \subseteq C$. Here $C = D$

(c) $P = \{3, 7, 9\}$ is not a subset of $Q = \{2, 4, 6, 7, 8, 9\}$ as $3 \notin Q$.

Note : If A is not a subset of B, we write

$$A \nsubseteq B$$

4.3.6 Universal set

In any application of set theory, all the sets under investigation are likely to be considered as subsets of a particular set. The set is called the ~~Universal~~ Universal set.

Some examples of universal set are given below:

(a) When a six-faced die is thrown and the point appearing is considered, the universal set $S = \{1, 2, 3, 4, 5, 6\}$, because, any discussion concerning the point obtained in any throw will related to the elements of S. For instance, 'even number of point' is the set $\{2, 4, 6\}$ that is a subset of S.

(b) In a study of income of Indians, the set of all figures of income of the people in India is the universal set, and the income of different groups or classes of people will form subsets of the universal set.

4.3.7 Disjoint Set

Sets A and B are said to be disjoint if they have no common element.

Examples (a) Let $A = \{1, 3, 5, 7\}$ $B = \{2, 4, 6, 8\}$. Then A and B are disjoint sets.

(b) $A = \{2, 4, 6\}$ and $B = \{1, 2, 3, 5\}$ are not disjoint as A and B has common element 2.

4.3.8 Family (class) of Sets

A set whose elements are themselves sets is called a family of sets of class of sets

Examples: (a) Let $A = \{2, 3\}$, $B = \{1, 4\}$, $C = \{5\}$

and $D = \phi$. Then $\{A, B, C, D\}$ is a family of sets.

(b) The set $\{\{a, b\}, \{c, d\}, \{e\}\}$ is a family of sets.

4.3.9 Power set

The family of sets that contains all the subsets of a set A as its elements, is called the power set of A. If the set A is finite and contains n elements, then there are 2^n possible subsets of A including the null set ϕ and the set A itself. The power set of A will then contain 2^n elements.

Examples: Let $A = \{3, 5\}$. Its possible subsets are ϕ, $\{3\}$, $\{5\}$ and $\{3, 5\}$. So, the power set of A is $\{\phi, \{3\}, \{5\}, \{3, 5\}\}$. It has $2^2 = 4$ elements

(b) Let $T = \{a, b, c\}$ Then the power set of T is $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

It has $2^3 = 8$ elements.

4.3.10 Product set (or Cartesian product of two sets)

Let A and B be two sets. We define the Cartesian product of A and B, denoted by $A \times B$, (It is read as A cross B) ~~is defined~~ as

$$A \times B = \{(a, b) : a \in A, b \in B\}$$ where

(a, b) is an ordered pair in the sense it is a pair where a is written first then

b is written second, they are ~~sp~~ seperated by a comma and ~~d~~ enclosed in brackets ( ). Two ordered pairs (a, b) and (c, d) are equal if $a = c$, $b = d$
$(3, 8)$ and $(8, 3)$ are different ordered pair

In particular, $A \times A = \{(a, b) : a \in A, b \in A\}$

Examples (a) Let $A = \{a, b\}$ $B = \{1, 2, 3\}$

Then $A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$B \times A = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

Each of $A \times B$ and $B \times A$ ~~have~~ has 6 elements,
$A \times B \neq B \times A$.

(b) Let $A = \{1, 2\}$
Then $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

1.4 Venn Diagram

It is often found convenient to illustrate the relationship between sets by using pictorial representation. Such a diagram is known as Venn-Euler Diagram or simply Venn Diagram.
A universal set S is generally represented geometrically by ~~a~~ a set of points inside a ~~circle~~ rectangle. Any sets A and B are represented by points inside ~~a~~ circles located within the rectangle.