The p distinct symbols can be arranged among themselves in $p!$ different ways. Thus, if this change is made in each of x permutations, we have $p! \cdot x$ permutations where q number of B's and r number of C's are there and the rest of them are all distinct.

Again, consider one of the $p! \cdot x$ permutions and replace q number of B's by q distinct symbols $B_1, B_2, \ldots, B_q$. The q distinct symbols can be arranged in $q!$ ~~ways~~ different ways. Thus, making this change in each of the $p! \cdot x$ permutations, we have $q! \cdot p! \cdot x$ permutations, where there are r number of C's and rest of them are all distinct. Proceeding similarly, replacing r number of C's by r distinct symbols $C_1, C_2, \ldots, C_r$ and permuting them, we have $r! \cdot q! \cdot p! \cdot x$ permutations, where all symbols are distinct. But we know that the number of permutations of n different symbols is $n!$.

Hence $\quad r! \cdot q! \cdot p! \cdot x = n!$

or, $\quad x = \dfrac{n!}{p!\,q!\,r!}$

<u>Corollary 1.3.3</u> The number of permutations of n objects taken all together, the items are not all different is

$$\frac{n!}{p_1!\,p_2!\cdots p_k!}$$ where p_1 are alike of one kind, p_2 objects are alike of second kind, $\ldots$, p_k objects are alike of kth kind.

<u>Result 1.3.4</u> From n distinct objects, r objects are taken at a time in which any object can be repeated up to r times,

the total number of possible arrangements is n^r

Proof: The result can be proved in the same way as that of Result 1.3.1 considering the repetition is allowed.

The number of ways the first place can be filled up is n. Since the repetition is allowed, the number of ways of filling up the second place is also n because the same object is available in the second place which already ~~of~~ occupies the first place. Thus, the first two places can be filled up together in $n \times n = n^2$ ways.

Proceeding similarly, the rth place can be filled up in n ways; since each of the objects can be repeated up to r times. Hence the total number of ways of filling up r places simultaneously is $n \times n \times n \cdots \times n$ (r times) $= n^r$.

Result 1.3.5 The number of permutations of n distinct objects taken r at a time in which q particular objects never occurs is ${}^{n-q}P_r$ where $r + q \leq n$.

Result 1.3.6 The number of permutations of n distinct objects taken r at a time in which q particular objects are always present is ${}^{n-q}P_{r-q} \times {}^{r}P_{q}$ where $q < r \leq n$

Result 1.3.7 The number of permutations of n distinct objects taken r at a time in which q particular objects come together in a given order is $(r-q+1) \times {}^{n-q}P_{r-q}$

Result 1.3.8 The number of permutation of n distinct objects taken r at a time in which q particular objects are placed in q given places

(a) in a definite order $^{n-q}P_{r-q}$

(b) in any order $q! \times {}^{n-q}P_{r-q}$

Example 1 Find the value of $^{8}P_{5}$

Solution: $^{8}P_{5} = \dfrac{8!}{(8-5)!} = \dfrac{8!}{3!} = \dfrac{8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1}$

$= 8\times7\times6\times5\times4 = 6720$

Example 2 Find the value of n when $^{n+1}P_{4} : {}^{n-1}P_{3} = 72 : 5$

Solution: Here $^{n+1}P_{4} = \dfrac{(n+1)!}{(n+1-4)!} = \dfrac{(n+1)!}{(n-3)!}$

and $^{n-1}P_{3} = \dfrac{(n-1)!}{(n-1-3)!} = \dfrac{(n-1)!}{(n-4)!}$

So, we have, from the given condition

$$\frac{(n+1)!}{(n-3)!} : \frac{(n-1)!}{(n-4)!} = 72 : 5$$

or, $$\frac{(n+1)!}{(n-3)!} \times \frac{(n-4)!}{(n-1)!} = \frac{72}{5}$$

or, $$\frac{n(n+1)}{n-3} = \frac{72}{5}$$

or, $$5n(n+1) = 72(n-3)$$

or, $$5n^2 + 5n - 72n + 216 = 0$$

or, $5n^2 - 67n + 216 = 0$

or, $5n^2 - 40n - 27n + 216 = 0$

or, $5n(n-8) - 27(n-8) = 0$

or, $(5n-27)(n-8) = 0$

or, $n = 8$ (since n is an integer)

Example 3 How many three-digit numbers can be formed with the digits 1, 2, 3, 4 and 5.

Solution: There are 5 digits – 1, 2, 3, 4 and 5. We have to form 3-digit numbers using these five digits. Thus, 3 digits numbers can be formed by taking any three out of the given five digits.

So, the required number of three digits

$$= {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$$

Example 4 In how many ways can the letters of the word 'BUSINESS' be arranged in following cases?

(a) There is no restriction.

(b) The vowels always occupy the odd places.

Solution: There are 8 letters in the word BUSINESS that includes 3 S. Therefore, the total number of arrangements

$$= \frac{8!}{3!} = 8 \times 7 \times 6 \times 5 \times 4 = 6720$$

(b) There are 4 odd places (first, third, fifth and seventh). Thus, four odd places can be occupied by 3 vowels (E, I, U)