The regions bounded by the circles overlap each other when A and B have some common elements shown in Figure 1(a). Disjoint sets are represented by non-overlapping circular regions shown in Figure 1(b)

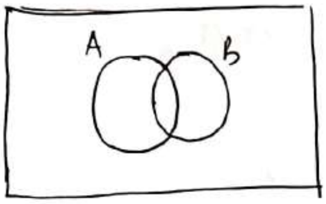


Figure 1(a)

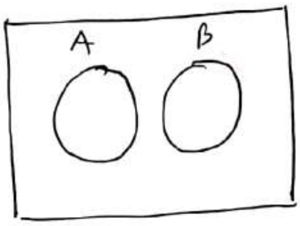
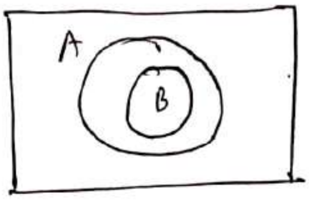


Figure 1(b)

If A is a subset of B, the region for A is shown wholly enclosed within the region for B as shown in Fig 1(c)



4.5 Set operations

In algebra, we ~~know~~ know the operations of addition, subtraction, multiplication etc. with numbers. For example, using the numbers 3 and 7, we get new number $3+7=10$, $3-7=-4$, $3\times 7=21$ etc. Similarly operations on sets obeying certain rules can be defined to form new sets. Broadly, there are four operations. They are

(i) Union
(ii) Intersection
(iii) Complement
(iv) Difference

4.5.1 Union of two sets

The union of sets A and B is defined

as (denoted by $A \cup B$)

$A \cup B = \{x : x \in A \text{ or } x \in B\}$. It is read as 'A union B'.

In venn diagram, it is shown by the shaded region in Figure 1(d)

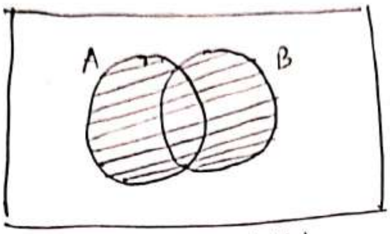


Figure 1(d)

Let $A = \{1, 3, 4\}$, $B = \{2, 3, 4, 5\}$

Then $A \cup B = \{1, 2, 3, 4, 5\}$

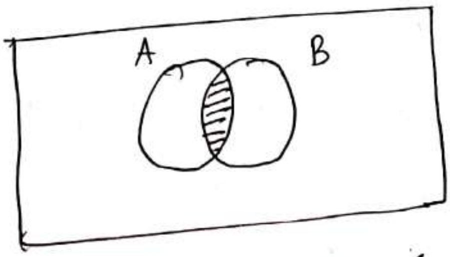
If $A = \{1, 2\}$ $B = \{3, 4\}$

Then $A \cup B = \{1, 2, 3, 4\}$

The intersection of sets A and B is defined as (denoted by $A \cap B$ and read as 'A intersection B')

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

In venn Diagram, it is shown by the shaded region as shown in the figure 1(e)



Let $A = \{1, 3, 4\}$, $B = \{2, 3, 4, 5\}$

Then $A \cap B = \{3, 4\}$

If $A = \{1, 2\}$ and $B = \{3, 4\}$

Then $A \cap B = \phi$, the null set.

For any set A, the complement of A, denoted by A^c and read as 'A complement', is defined as

$$A^c = \{x : x \notin A\}$$

To find A^c, you have to know what the universal set U is. In the venn diagram, it is denoted by the shaded region as shown in Figure 1(f). Here, as we know earlier, the rectangle shown is the universal set

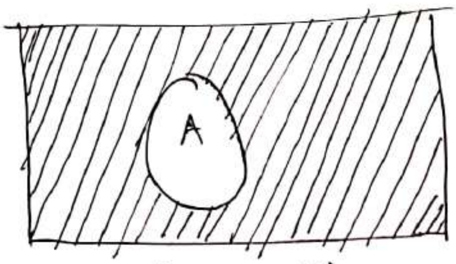


Figure 1(f)

Let $A = \{1, 2\}$ and the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Then $A^c = \{3, 4, 5, 6, 7, 8, 9, 10\}$

If the universal set is the set of all natural numbers $\mathbb{N}$, then

$$A^c = \{2, 3, 4, \ldots\}$$

The difference of two sets A and B is defined as (denoted by A−B, read as 'A minus B')

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

Let $A = \{1, 3, 5\}$ and $B = \{1, 3, 4, 6\}$

Then $A - B = \{5\}$

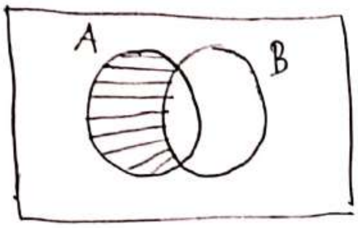
If $A = \{1\}$ $B = \{2, 3\}$

$A - B = A = \{1\}$

If $A = \{1, 3\}$ $B = \{1, 3, 4, 6, 7\}$

then $A - B = \phi$

It is shown by the shaded region in figure 1(g)



If U be the ~~universet~~ universal set and A and B are two subsets of U, then the following properties can be easily verified :

(i) $A \cap A = A$

(ii) $A \cup A = A$

(iii) $A \cap U = A$

(iv) $A \cup U = U$

(v) If $A \subseteq B$ then $A \cap B = A$
and $A \cup B = B$

(vi) $A \cap \phi = \phi$

(vii) $A \cup \phi = A$

(viii) $A \cup A^c = U$

(ix) $A \cap A^c = \phi$

For two disjoint sets, $A \cap B = \phi$

Worked Examples : Example 1 If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 8\}$ and $C = \{3, 4, 5, 6, 7\}$, then find

(i) $A \cap B$ (ii) $B \cup C$ (iii) $A \cap (B \cup C)$ (iv) $A \cup (B \cap C)$

Solution: (i) $A \cap B = \{x : x \in A \text{ and } x \in B\} = \{2, 4\}$