(ii) $B \cup C = \{x : x \in B \text{ or } x \in C\}$ ('or' is in the inclusive sense of i.e., or both included)

$= \{2, 3, 4, 5, 6, 7, 8\}$

(iii) $A \cap (B \cup C) = \{x : x \in A \text{ and } x \in B \cup C\}$

$= \{2, 3, 4\}$

(iv) $B \cap C = \{x : x \in B \text{ and } x \in C\}$

$= \{4, 5\}$

So, $A \cup (B \cap C) = \{x : x \in A \text{ or } x \in B \cap C\}$

$= \{1, 2, 3, 4, 5\}$

<u>Example 2</u> If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 3, 4, 6, 8\}$ verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Solution : $B \cup C = \{x : x \in B \text{ or } x \in C\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

So, L.H.S $= A \cap (B \cup C) = \{x : x \in A \text{ and } x \in B \cup C\}$

$= \{1, 2, 3, 4\}$ . . . (i)

Again, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

$= \{2, 4\}$

$A \cap C = \{x : x \in A \text{ and } x \in C\}$

$= \{1, 3, 4\}$

So, R.H.S $= (A \cap B) \cup (A \cap C)$

$= \{x : x \in A \cap B \text{ ~~and~~ or } x \in A \cap C\}$

$= \{1, 2, 3, 4\}$ . . . (ii)

So, From (i) and (ii) we see that L.H.S = R.H.S

Example 3 (a) If $A = \{1,2,3,4,6\}$, $B = \{2,4,5\}$, $C = \{3,4,5,7\}$,

find (i) $A-B$ (ii) $A-C$ (iii) $A-(B\cup C)$

Hence, verify that $A-(B\cup C) = (A-B)\cap(A-C)$

(b) If $S = \{1,2,3,4,5,6\}$ be the universal set, $A = \{1,3,4\}$

and $B = \{2,3,4,5\}$, find

(i) A' (ii) $(A\cup B)'$ (iii) $A\cap B'$ (iv) $A'\cap B'$ and verify that

(v) $(A\cup B)' = A'\cap B'$ where A' is A^c.

Solution: ~~A-B~~ (a) (i) $A-B = \{x : x\in A \text{ and } x\notin B\}$

$= \{1,3,6\}$

(ii) $A-C = \{x : x\in A \text{ and } x\notin C\}$

$= \{1,2,6\}$

(iii) ~~A-B∪C~~ $B\cup C = \{x : x\in B \text{ or } x\in C\}$

$= \{2,3,4,5,7\}$

So, $A-(B\cup C) = \{x : x\in A \text{ and } x\notin (B\cup C)\}$

$= \{1,6\}$

Now $(A-B)\cap(A-C) = \{x : x\in A-C \text{ and } x\in A-C\}$

$= \{1,6\}$

So, ~~A-B∪C~~ $A-(B\cup C) = (A-B)\cap(A-C)$

(b) (i) $A' = \{x\in S : x\notin A\} = \{2,5,6\}$

(ii) $A\cup B = \{x\in S : x\in A \text{ or } x\in B\}$

$= \{1,2,3,4,5\}$

So, $(A\cup B)' = \{x\in S : x\notin A\cup B\}$

$= \{6\}$

(iii) $B' = \{x \in S : x \notin B\} = \{1, 6\}$

So, $A \cap B' = \{x \in S : x \in A \text{ and } x \in B'\}$

$= \{1\}$

(iv) $A' \cap B' = \{x \in S : x \in A' \text{ and } x \in B'\}$

$= \{6\}$

(v) From (ii) and (iv), we have, $A' \cap B' = (A \cup B)' = \{6\}$

Example 4 If $A = \{2, 3, 5\}$ and $B = \{a, b\}$, find $A \times B$ and $B \times A$ and shows that $A \times B \neq B \times A$

Solution : $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$

$= \{(2, a), (3, a), (5, a), (2, b), (3, b), (5, b)\}$

$B \times A = \{(x, y) : x \in B \text{ and } y \in A\}$

$= \{(a, 2), (a, 3), (a, 5), (b, 2), (b, 3), (b, 5)\}$

Here $(2, a) \in A \times B$ but $(2, a) \notin B \times A$

Also, $(a, 3) \in B \times A$ but $(a, 3) \notin A \times B$

So, $A \times B \neq B \times A$

Example 5 If $A = \{2, 5\}$, $B = \{5, 6\}$, $C = \{6, 8\}$, show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution: $B \cup C = \{x : x \in B \text{ or } x \in C\} = \{5, 6, 8\}$

So, $A \times (B \cup C) = \{(x, y) : x \in A, y \in B \cup C\}$

$= \{(2, 5), (2, 6), (2, 8), (5, 5), (5, 6), (5, 8)\}$

Again, $A \times B = \{(x, y) : x \in A, y \in B\} = \{(2, 5), (2, 6), (5, 5), (5, 6)\}$

$A \times C = \{(x, y) : x \in A, y \in C\} = \{(2, 6), (2, 8), (5, 6), (5, 8)\}$

So, $(A \times B) \cup (A \times C) = \{(2,5),(2,6),(2,8),(5,5),(5,6),(5,8)\}$

As $A \times (B \cup C)$ and $(A \times B) \cup (A \times C)$ have the same elements

So, $A \times (B \cup C) = (A \times B) \cup (A \times C)$

4.6 Laws of Algebra of sets :

Under the operations of Union, Intersection and Complement, sets satisfy various laws, some of which have been noted earlier. These are referred to as Laws of Algebra of sets.

1. Commutative Laws
   (i) $A \cup B = B \cup A$
   (ii) $A \cap B = B \cap A$
2. Associative Laws
   (i) $A \cup (B \cup C) = (A \cup B) \cup C$
   (ii) $A \cap (B \cap C) = (A \cap B) \cap C$
3. Distributive Laws
   (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. De Morgan's Laws
   (i) $(A \cup B)' = A' \cap B'$ (A' is the complement of A)
   (ii) $(A \cap B)' = A' \cup B'$

The following laws shows the results of operation involving any set A, the universal set S and the null set ϕ and are quite obvious from a knowledge rules of the set operations

5. Idempotent Laws
   (i) $A \cup A = A$ (ii) $A \cap A = A$