$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

So, $(A \cup B)^c \subseteq A^c \cap B^c$

$y \in A^c \cap B^c \Rightarrow y \in A^c$ and $y \in B^c$

$\Rightarrow y \notin A$ and $y \notin B$

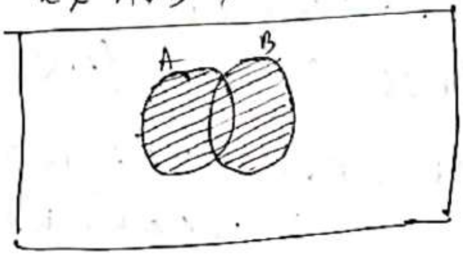
$\Rightarrow y \notin A \cup B$

$\Rightarrow y \in (A \cup B)^c$

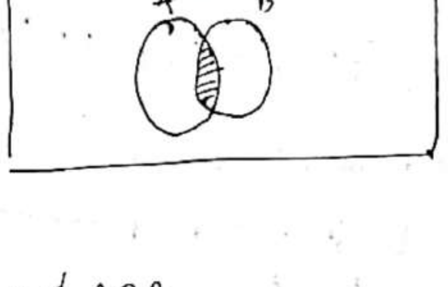
So, $A^c \cap B^c \subseteq (A \cup B)^c$

So, $(A \cup B)^c = A^c \cap B^c$

[Note: $x \in A \cup B \Rightarrow x$ is in the shaded region given below and $x \notin A \cup B \Rightarrow x$ is not in the shaded region, i.e, $x \notin A$ and $x \notin B$



Similarly $x \in A \cap B \Rightarrow x$ is in the shaded region given below and $x \notin A \cap B \Rightarrow x$ is not in the shaded region, i.e., $x \notin A$ or $x \notin B$.



]

(ii) $x \in (A \cap B)^c \Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

So, $(A \cap B)^c \subseteq A^c \cup B^c$

$y \in A^c \cup B^c \Rightarrow y \in A^c$ or $y \in B^c$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin A \cap B$

$\Rightarrow y \in (A \cap B)^c$

So, $A^c \cup B^c \subseteq (A \cap B)^c$

So, $(A \cap B)^c = A^c \cup B^c$

Example 10 Applying the laws of algebra of sets, show that

(i) $(A \cup B) \cap (A \cup B') = A$

(ii) $A \cup (A' \cap B) = A \cup B$

(iii) $A \cap (A \cup B) = A$, A' is the complement of A

Solution: (i) $(A \cup B) \cap (A \cup B') = A \cup (B \cap B')$ by Distributive Law

$= A \cup \phi$ by Complement Law

$= A$ by Identity Law

(ii) $A \cup (A' \cap B) = (A \cup A') \cap (A \cup B)$ by Distributive Law

$= S \cap (A \cup B)$ by Complement Law and S is the universal set

$= (A \cup B) \cap S$ by Commutative Law

$= A \cup B$ by Identity Law

(iii) $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$ by Distributive Law

$= A \cup (A \cap B)$

$= A$ (As $A \cup B = A$ if $B \subseteq A$)

Alternative proof: $A \cap (A \cup B) = (A \cup \phi) \cap (A \cup B)$ by Identity Law

$= A \cup (\phi \cap B)$ by Distributive Law

$= A \cup (B \cap \phi)$ by Commutative Law

$= A \cup \phi$ by Identity Law

$= A$ by Identity Law

Example 11 If $A \subseteq B$, prove that

(i) $A \cup B = B$

(ii) $A \cap B = A$

(iii) $A - B = \phi$

Solution : (i) $x \in A \cup B \Rightarrow x \in A$ or $x \in B \Rightarrow x \in B$ or $x \in B$ ($\because A \subseteq B$)

$\Rightarrow x \in B$. So, $A \cup B \subseteq B$

Again $B \subseteq A \cup B$ (by definition)

So, $A \cup B = B$

(ii) Now, $A \cap B \subseteq A$ (by definition)

and $x \in A \Rightarrow x \in A$ and $x \in A$

$\Rightarrow x \in A$ and $x \in B$ ($\because A \subseteq B$)

$\Rightarrow x \in A \cap B$

So, $A \subseteq A \cap B$. So, $A \cap B = A$

(iii) $x \in A - B \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in B$ and $x \in B'$ ($\because A \subseteq B$, and B' is the complement of B)

$\Rightarrow x \in B \cap B'$

$\Rightarrow x \in \phi$ ($\because B \cap B' = \phi$)

So, $A - B \subseteq \phi$

Now ϕ is a subset of any set. So,

$\phi \subseteq A - B$

So, $A - B = \phi$

Difference of sets (Important results)

1. $A - B = A \cap B' = B' - A'$ (A' is the complement of A)

2. (i) $A - (B \cup C) = (A - B) \cap (A - C)$

(ii) $A-(B\cap C) = (A-B)\cup(A-C)$

3\. (i) $(A\cup B)-C = (A-C)\cup(B-C)$

(ii) $(A\cap B)-C = (A-C)\cap(B-C)$

4\. $A\cap(B-C) = (A\cap B)-C = (A\cap B)-(A\cap C)$

Example 12 Prove that

(i) $A-B = A\cap B'$

(ii) $A-B = B'-A'$

Solution: $x\in A-B \Rightarrow x\in A$ and $x\notin B$

$\Rightarrow x\in A$ and $x\in B'$

$\Rightarrow x\in A\cap B'$

So, $A-B \subseteq A\cap B'$ --- (1)

Now, $y\in A\cap B' \Rightarrow y\in A$ and $y\in B'$

$\Rightarrow y\in A$ and $y\notin B$

$\Rightarrow y\in A-B$

So, $A\cap B' \subseteq A-B$ --- (2)

Hence from (1) and (2), $A-B = A\cap B'$

(ii) $x\in A-B \Rightarrow x\in A$ and $x\notin B$

$\Rightarrow x\notin A'$ and $x\in B'$

$\Rightarrow x\in B'$ and $x\notin A'$

$\Rightarrow x\in B'-A'$

So, $A-B \subseteq B'-A'$ ... (1)

Now, $y\in B'-A' \Rightarrow y\in B'$ and $y\notin A'$

$\Rightarrow y\notin B$ and $y\in A$

$\Rightarrow y\in A$ and $y\notin B$