$\Rightarrow y \in A-B$

So, $B'-A' \subseteq A-B$ . . . (2)

Hence from (1) and (2) $A-B = B'-A'$

Example 13 Applying the laws of set algebra, show that

(i) $A \cup (B-A) = A \cup B$

(ii) $A \cap (B-A) = \phi$

(iii) $A-(A-B) = A \cap B$

(iv) $A-(B-A) = A$

Solution (i) $A \cup (B-A) = A \cup (B \cap A')$

$= (A \cup B) \cap (A \cup A')$ by Distributive Law

$= (A \cup B) \cap S$ by Complement Law (S is the universal set)

$= (A \cup B)$ by Identity Law

(ii) $A \cap (B-A) = A \cap (B \cap A')$

$= A \cap (A' \cap B)$ by Commutative Law

$= (A \cap A') \cap B$ by Associative Law

$= \phi \cap B$ by Complement Law

$= B \cap \phi$ by Commutative Law

$= \phi$ by Identity Law

(iii) $A-(A-B) = A-(A \cap B')$ since $A-B = A \cap B'$

$= A \cap (A \cap B')'$ since $A-C = A \cap C'$

$= A \cap (A' \cup B)$ by De Morgan's Law and Complement Law $(B')' = B$

$= (A \cap A') \cup (A \cap B)$ by Distributive Law

$= \phi \cup (A \cap B)$ by Complement Law

$= A \cap B$ by Identity Law

(iv) $A-(B-A) = A-(B\cap A')$ since $B-A = B\cap A'$

$= A\cap (B\cap A')'$ since $A-C = A\cap C'$

$= A\cap (B'\cup A)$ by De Morgan's Law and Complement Law $(A')' = A$

$= A\cap (A\cup B')$ by Commutative Law

$= (A\cup \phi)\cap (A\cup B')$ by Identity Law

$= A\cup (\phi\cap B')$ by Distributive Law

$= A\cup \phi$ by Identity Law

$= A$ by Identity Law

Example 14 Prove that

(i) $A-(B\cup C) = (A-B)\cap (A-C)$

(ii) $A-(B\cap C) = (A-B)\cup (A-C)$

Solution: (i) $x\in A-(B\cup C) \Rightarrow x\in A$ and $x\notin (B\cup C)$

$\Rightarrow x\in A$ and $(x\notin B$ and $x\notin C)$

$\Rightarrow (x\in A$ and $x\notin B)$ and $(x\in A$ and $x\notin C)$

$\Rightarrow x\in A-B$ and $x\in A-C$

$\Rightarrow x\in (A-B)\cap (A-C)$

So, $A-(B\cup C) \subseteq (A-B)\cap (A-C)$ ... (1)

Also, $y\in (A-B)\cap (A-C)$

$\Rightarrow y\in (A-B)$ and $y\in (A-C)$

$\Rightarrow (y\in A$ and $y\notin B)$ and $(y\in A$ and $y\notin C)$

$\Rightarrow y\in A$ and $(y\notin B$ and $y\notin C)$

$\Rightarrow y\in A$ and $y\notin B\cup C$

$\Rightarrow y\in A-(B\cup C)$

So, $(A-B)\cap (A-C) \subseteq A-(B\cup C)$ ... (2)

Hence from (1) and (2), $A-(B\cup C) = (A-B)\cap (A-C)$

(ii) $x \in A-(B\cap C) \Rightarrow x\in A$ and $x \notin B\cap C$

$\Rightarrow x\in A$ and $(x\notin B$ or $x \notin C)$

$\Rightarrow (x\in A$ and $x\notin B)$ or $(x\in A$ and $x\notin C)$

$\Rightarrow x\in$ ~~and~~ $A-B$ or $x\in A-C$

$\Rightarrow x\in (A-B)\cup(A-C)$

So, $A-(B\cap C) \subseteq (A-B)\cup(A-C)$ ... (1)

Now, $y\in (A-B)\cup(A-C) \Rightarrow y\in A-B$ or $y\in A-C$

$\Rightarrow (y\in A$ and $y\notin B)$ or $(y\in A$ and $y\notin C)$

$\Rightarrow y\in A$ and $(y\notin B$ or $y\notin C)$

$\Rightarrow y\in A$ and $y\notin B\cap C$

$\Rightarrow y\in A-$ ~~$(B\cup C)$~~ $(B\cap C)$

So, $(A-B)\cup(A-C) \subseteq A-(B\cap C)$ ... (2)

Hence from (1) and (2), we have

~~$A-B\cap A-B\cap A$~~

$A-(B\cap C) = (A-B)\cup(A-C)$

Example 15 Prove that

(i) $(A\cup B)-C = (A-C)\cup(B-C)$

(ii) $(A\cap B)-C = (A-C)\cap(B-C)$

Solution: (i) $x\in (A\cup B)-C \Rightarrow x\in A\cup B$ and $x\notin C$

$\Rightarrow (x\in A$ or $x\in B)$ and $x\notin C$

$\Rightarrow (x\in A$ and $x\notin C)$ or $(x\in B$ and $x\notin C)$

$\Rightarrow x\in (A-C)$ or $x\in B-C$

$\Rightarrow x\in (A-C)\cup(B-C)$

So, $(A\cup B)-C \subseteq (A-C)\cup(B-C)$ ... (1)

Now, $y\in (A-C)\cup(B-C) \Rightarrow y\in A-C$ or $y\in B-C$

$\Rightarrow (y \in A \text{ and } y \notin C) \text{ or } (y \in B \text{ and } y \notin C)$

~~$y \in A$ and ($y \notin$ or y~~

$\Rightarrow (y \in A \text{ or } y \in B) \text{ and } y \notin C$

$\Rightarrow y \in A \cup B \text{ and } y \notin C$

$\Rightarrow y \in (A \cup B) - C$

So, $(A-C) \cup (B-C) \subseteq (A \cup B) - C$ --- (2)

Hence from (1) and (2), we have

$$(A \cup B) - C = (A-C) \cup (B-C)$$

(ii) $x \in (A \cap B) - C \Rightarrow x \in A \cap B \text{ and } x \notin C$

$\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \notin C$

$\Rightarrow (x \in A \text{ and } x \notin C) \text{ and } (x \in B \text{ and } x \notin C)$

$\Rightarrow x \in A - C \text{ and } x \in B - C$

$\Rightarrow x \in (A-C) \cap (B-C)$

So, $(A \cap B) - C \subseteq (A-C) \cap (B-C)$ --- (1)

Now, $y \in (A-C) \cap (B-C) \Rightarrow y \in A - C \text{ and } y \in B - C$

$\Rightarrow (y \in A \text{ and } y \notin C) \text{ and } (y \in B \text{ and } y \notin C)$

$\Rightarrow (y \in A \text{ and } y \in B) \text{ and } y \notin C$

$\Rightarrow y \in A \cap B \text{ and } y \notin C$

$\Rightarrow y \in (A \cap B) - C$

So, $(A-C) \cap (B-C) \subseteq (A \cap B) - C$ --- (2)

Hence from (1) and (2), we have

$$(A \cap B) - C = (A-C) \cap (B-C)$$