(ii) $(x,y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in B \cap C$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow$ ~~x∈A~~ $(x,y) \in A \times B$ and $(x,y) \in A \times C$

$\Rightarrow (x,y) \in (A \times B) \cap (A \times C)$

So, $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ --- (1)

$(a,b) \in (A \times B) \cap (A \times C)$

~~⇒ a∈A×B and~~

$\Rightarrow (a,b) \in A \times B$ and $(a,b) \in A \times C$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow a \in A$ and $(b \in B$ and $b \in C)$

$\Rightarrow a \in A$ and $b \in B \cap C$

$\Rightarrow (a,b) \in A \times (B \cap C)$

So, $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ --- (2)

So, from (1) and (2), we have

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(x,y) \in A \times (B - C)$

$\Rightarrow x \in A$ and $y \in B - C$

$\Rightarrow x \in A$ and $(y \in B$ and $y \notin C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C)$

$\Rightarrow (x,y) \in A \times B$ and $(x,y) \notin (A \times C)$

$\Rightarrow (x,y) \in (A \times B) - (A \times C)$

So, $A \times (B - C) \subseteq (A \times B) - (A \times C)$ --- (1)

$(a,b) \in (A \times B) - (A \times C)$

$\Rightarrow (a,b) \in A \times B$ and $(a,b) \notin A \times C$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \notin C)$

$\Rightarrow a \in A$ and $(b \in B$ and $b \notin C)$

$\Rightarrow a \in A$ and $b \in B - C$

$\Rightarrow (a,b) \in A \times (B - C)$

So, $(A \times B) - (A \times C) \subseteq A \times (B - C)$ --- (2)

From (1) and (2) we have

$$A \times (B - C) = (A \times B) - (A \times C)$$

Example 20 Prove that.

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

Solution: $(x,y) \in (A \times B) \cap (C \times D)$

~~$x \in (A \times B)$ and~~

$\Rightarrow (x,y) \in (A \times B)$ and $(x,y) \in (C \times D)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in C$ and $y \in D)$

$\Rightarrow (x \in A$ and $x \in C)$ and $(y \in B$ and $y \in D)$

$\Rightarrow x \in A \cap C$ and $y \in B \cap D$

$\Rightarrow (x,y) \in (A \cap C) \times (B \cap D)$

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So, $(A\times B)\cap(C\times D) \subseteq (A\cap C)\times(B\cap D)$ - - - (1)

$(a,b)\in(A\cap C)\times(B\cap D)$

$\Rightarrow$ $a\in(A\cap C)$ and $b\in B\cap D$

$\Rightarrow$ $(a\in A$ and $a\in C)$ and $(b\in B$ and $b\in D)$

$\Rightarrow$ $(a\in A$ and $b\in B)$ and $(a\in C$ and $b\in D)$

$\Rightarrow$ $(a,b)\in A\times B$ and $(a,b)\in C\times D$

$\Rightarrow$ $(a,b)\in(A\times B)\cap(C\times D)$

So, $(A\cap C)\times(B\cap D) \subseteq (A\times B)\cap(C\times D)$ -- (2)

From (1) and (2), we get

$$(A\times B)\cap(C\times D) = (A\cap C)\times(B\cap D)$$

4.8 Number of Elements in a set

Let A be a finite set. We shall denote the number of elements of A by the symbol $n(A)$.

~~When two or more sets are given~~

When two or more finite sets are given, new sets may be derived by the operations of Union, intersection, difference and complement on those sets. Here we shall show some relations among the number of elements in the sets and their subsets.

Example 21 If A and B are disjoint finite sets, prove that

$$n(A\cup B) = n(A)+n(B)$$

Solution: Since A and B are disjoint sets, there is no element common to A and B. So, set $A \cup B$ includes all the elements of A and B. So, the number of elements of $A \cup B$ is the sum of the elements of A and the elements of B. So,

$$n(A \cup B) = n(A) + n(B)$$

Example 22 For two finite sets A and B, show that (i) $n(A) = n(A \cap B) + n(A \cap B')$

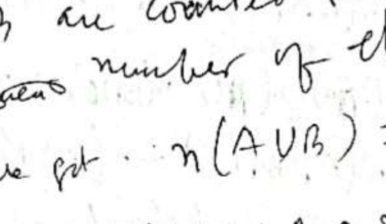
(ii) $n(B) = n(A \cap B) + n(A' \cap B)$

Solution: (i) Since $A \cap B$ and $A \cap B'$ are disjoint and $(A \cap B) \cup (A \cap B') = A$

So, $n(A) = n(A \cap B) + n(A \cap B')$

Similarly, (ii) $n(B) = n(A \cap B) + n(A' \cap B)$

Note: With the help of Venn diagram, we get $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, if A and B are finite sets.



For counting of elements of $A \cup B$, if we take all elements of A and B then the common elements of A and B are counted twice. So, we subtract the number of elements of $A \cap B$

So, we get $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

When $A \cap B = \phi$ i.e., A and B are disjoint, we get $n(A \cup B) = n(A) + n(B)$