Example 23. For any three finite sets A, B and C, prove that

$n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$

Solution: We know that for any two sets, A and B

$$n(A\cup B) = n(A)+n(B)-n(A\cap B)$$

Now $A\cup B\cup C = A\cup(B\cup C) = A\cup X$ where $X = B\cup C$

So, $n(A\cup X) = n(A)+n(X)-n(A\cap X)$

$= n(A)+n(B\cup C)-n(A\cap(B\cup C))$

$= n(A)+n(B)+n(C)-n(B\cap C)-n((A\cap B)\cup(A\cap C))$

$= n(A)+n(B)+n(C)-n(B\cap C)-\left(n(A\cap B)+n(A\cap C)-n((A\cap B)\cap(A\cap C))\right)$

$= n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)-n(A\cap B\cap C)$

Note: In particular, if A, B, C are mutually disjoint ~~finite~~ finite sets, then $A\cap B$, $A\cap C$, $B\cap C$ and $A\cap B\cap C$ are all null sets. So, $n(A\cup B\cup C) = n(A)+n(B)+n(C)$

Important Results

For finite sets A, B, C,

1. $n(A\cup B) = n(A)+n(B)-n(A\cap B)$
   $= n(A\cap B)+n(A\cap B')+n(A'\cap B)$

2. $n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$
   $= n(A\cap B\cap C)+n(A\cap B\cap C')+n(A\cap B'\cap C)$
   $+n(A'\cap B\cap C)+n(A\cap B'\cap C')+n(A'\cap B\cap C')+n(A'\cap B'\cap C)$

3. (i) $n(A\cap B') = n(A)-n(A\cap B)$

4. (ii) $n(A' \cap B) = n(B) - n(A \cap B)$

4. (i) $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$

(ii) $n(A \cap B' \cap C) = n(A \cap C) - n(A \cap B \cap C)$

(iii) $n(A' \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$

5. $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$n(A' \cap B \cap C') = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$n(A' \cap B' \cap C) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Example 24. In a class of 75 students, 48 read Economics, 33 read Mathematics and 19 read both subjects. How many have taken (i) none of these subjects (ii) Economics but not Mathematics?

Solution: Let A denotes the set of students who read Economics and B the set of students who read Mathematics. We are given total number of students $N = 75$, $n(A) = 48$, $n(B) = 33$ and $n(A \cap B) = 19$

(i) The number of students who read at least one of the two subjects is

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 48 + 33 - 19 = 62$$

Since there are 75 students in all, the number of students who do not read any of the two subjects is $75 - 62 = 13$

(ii) The number of students who read Economics but not Mathematics is

$$n(A \cap B') = n(A) - n(A \cap B)$$
$$= 48 - 19 = 29$$

<u>Example 25</u> Out of 247 Candidates who failed in the C.A. Entrance examination, it was revealed that 128 failed in English, 87 in paper III and 134 in the aggregate. 31 failed in English and in paper III, 54 failed in aggregate and in English, 30 failed in the aggregate and in paper III. Find how many candidates

(i) failed in all the three

(ii) failed in English but not in Paper III

(iii) failed in the aggregate but not in English

(iv) ~~in paper III~~ failed in paper III but not in the aggregated or in English

(v) failed in the aggregate or in English but not in paper III)

<u>Solution</u>: Let A, B, C be the sets of candidates who faild in English, Paper III and aggregate respectively.

Given $n(A \cup B \cup C) = 247$, $n(A) = 128$, $n(B) = 87$, $n(C) = 134$,

$n(A \cap B) = 31$, $n(A \cap C) = 54$, and $n(B \cap C) = 30$

we have to find (i) $n(A \cap B \cap C)$, (ii) $n(A \cap B')$,

(iii) ~~n(A'...)~~ $n(A' \cap C)$, (iv) $n(B \cap (A \cup C)')$

and (v) $n((A\cup C)\cap B')$

(i) $n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$

or, $247 = 128+87+134-31-54-30+n(A\cap B\cap C)$

or, $247 = 234+n(A\cap B\cap C)$

So, $n(A\cap B\cap C) = 247-234 = 13$

(ii) $n(A\cap B') = n(A)-n(A\cap B)$

$= 128-31 = 97$

(iii) $n(A'\cap C) = n(C)-n(A\cap C) = 134-54 = 80$

(iv) $B\cap(A\cup C)' = B\cap(A'\cap C') = B\cap A'\cap C' = A'\cap B\cap C'$

$n(A'\cap B\cap C') = n(B)-n(A\cap B)-n(B\cap C)+n(A\cap B\cap C)$

$= 87-31-30+13$

$= 39$

(v) $(A\cup C)\cap B' = (A\cap B')\cup(C\cap B')$

$= (A\cap B')\cup(B'\cap C)$

$n((A\cup C)\cap B') = n(A\cap B')+n(B'\cap C) - n(A\cap B'\cap C)$

$= [n(A)-n(A\cap B)]+[n(C)-n(B\cap C)] - (n(A\cap C)-n(A\cap B\cap C))$

$= (128-31)+(134-30)-(54-13)$

$= 97+104-41 = 160$

<u>Example 26</u> The following report was submitted