by a field worker in market research. "Of 1000 people interviewed 811 liked chocolates, 752 liked toffee, and 418 liked boiled sweets. 570 liked chocolates and toffee, 356 liked chocolates and boiled sweets and 348 liked toffee and boiled sweets. 297 liked all three". Show that the report is incorrect.

Solution: Let A, B, C denote the sets of people who liked chocolates, toffee and boiled sweets, respectively. We are given that the total number of people is

$$N = 1000,\ n(A) = 811,\ n(B) = 752,\ n(C) = 418$$

$$n(A \cap B) = 570,\ n(A \cap C) = 356,\ n(B \cap C) = 348$$

and $n(A \cap B \cap C) = 297$

Using these data, the number of people who liked At least one of these three sweets is

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

$$= 811 + 752 + 418 - 570 - 356 - 348 + 297$$

$$= 2278 - 1274 = 1004$$

The number who did not like any of the three sweets is therefore $= n(A' \cap B' \cap C') = N - n(A \cup B \cup C)$

$$= 1000 - 1004 = -4$$

But the number of elements of any set can not be negative. Hence the given information is incorrect.

Exercises 1. If $A=\{1,3,5,7\}$, $B=\{2,3,4,5\}$, $C=\{2,4,6,8\}$, $D=\{4,5,6\}$, find

(i) $C \cup D$ (ii) $A \cap B$ (iii) $B \cup (C \cup D)'$ (iv) $A \cap (B \cup C)$

2. If $A=\{1,2,3,4\}$, $B=\{2,4,5,8\}$, $C=\{3,4,5,6,7\}$, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

3. Prove that

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

4. Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$

5. If $A=\{1,4\}$, $B=\{4,5\}$, $C=\{5,7\}$, find

(i) $(A \times B) \cup (B \times C)$

(ii) $(A \times B) \cap (B \times C)$

6. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

7. In a city, three daily newspaper A, B, C are published. 42% of the people in that read A; 51% read B; 68% read C; 30% read A and B; 28% read B and C; 36% read A and C; 8% do not read any of the three newspapers. Find the number of persons who read all three newspapers, using the above result.

8. In a survey of 100 students, it was found 50 used the College Library books, 40 had their own books and 30 used borrowed books, 20 used both College library and their own books, 15 used their own books and borrowed books, whereas 10 used College library books borrowed books. Assuming that each student uses an borrowed book either College library books or their own or borrowed books, find the number of students using books from all three sources.

6. Compound Interest and Annuities

6.1 Simple AP and GP Series

A series of observations is said to be in Arithmetic Progression (AP) if the consecutive values have a common difference.

Examples are (i) 1, 2, 3, 4, ...

(ii) 5, 8, 11, 14

(iii) 7, 5, 3, 1, −1, −3, −5, ...

The AP series is algebraically represented by

$$a, (a+d), (a+2d), (a+3d), \ldots \quad \ldots (1)$$

where a = initial term and d = common difference. The nth term (t_n) and the sum (S_n) upto n terms of the AP series are given by

$$t_n = a + (n-1)d \quad \text{and} \quad S_n = \frac{n}{2}\{2a + (n-1)d\} \quad \cdots (2)$$

Example 2. A series of observation is said to be in Geometric progression (GP), if the consecutive values have a common ratio.

Examples are (i) 5, 10, 20, 40, 80, 160, ...

(ii) 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, ...

The GP series is algebraically represented by

$$a, ar, ar^2, ar^3, \ldots \quad \cdots (3)$$

where a = initial term and r = common ratio. The nth term (t_n') and the sum (S_n') upto n terms of the GP series are given by

$$t_n' = ar^{n-1} \quad \text{and} \quad S_n' = \frac{a(1-r^n)}{1-r} \quad \cdots (4)$$

Example 1 Two men X and Y started working for a certain company at similar jobs on January 1, 1950. X asked for an initial salary of Rs. 300 with an annual increment of Rs. 30. Y asked for an initial salary of Rs. 200 with a rise of Rs. 15 every six months. What are their respective salaries on December 31, 1959, assuming that the arrangements remained unaltered? Find the total amount paid to each of them as salary during the period.

Solution: The initial monthly salary of X is