in ${}^4P_3$ ways. Again five more places can be filled by 5 consonants (B, N, S, S, S) of which 3 are alike in $\frac{5!}{3!}$ ways. Therefore the required number of arrangements

$$= {}^4P_3 \times \frac{5!}{3!} = \frac{4!}{(4-3)!} \times 5 \times 4 = 4\times3\times2\times1 \times 5\times4$$

$$= 24 \times 20 = 480$$

Example 5 Find the number of arrangements of the letters of the word 'STATISTICS' if

(a) the vowels appear together, and

(b) order of the vowels remains unchanged.

Solution: There are 10 letters in the word STATISTICS, where S occurs 3 times, T occurs 3 times and I occurs 2 times.

(a) We have 3 vowels (A, I, I) in the word. If the vowels appear together, we can consider AII as a single letter. Under this assumption, there are 8 letters including 3 S and 3 T. So they can be arranged in $\frac{8!}{3!\,3!}$ ways. Further, for each of these arrangements, 3 vowels can be arranged among themselves in $\frac{3!}{2!}$ ways. Therefore, the total number of arrangements when vowels appear together $= \frac{8!}{3!\,3!} \times \frac{3!}{2!} = 3360$

(b) Here the order of the vowels remains unchanged. That is, vowels can not arrange among themselves.

So we consider the three vowels as alike letters.

Therefore, the total number of arrangements with the order of the vowels remains unchanged (i.e., 3S, 3T and 3 vowels)

$$= \frac{10!}{3!3!3!} = 16800$$

Example 6 How many odd numbers less than 1000 can be formed using digits 0, 2, 3, 5 where repitions of digits are allowed?

Solution: The odd number less than 1000 can be 3-digit, 2-digit and 1-digit numbers.

First we compute the number of 3-digit odd numbers. The number of ways to fill in unit's place is 2 because either 3 or 5 can occupy the unit's place. The ten's place can be filled up by any one of the 4 digits 0, 2, 3 and 5. So it can be done in 4 ways. Further, the hundred's place can be filled up by the digits 2, 3 and 5, which can be done in 3 ways.

Then the number of 3-digit odd numbers

$$= 2 \times 3 \times 4 = 24$$

Similarly, to get the 2-digit odd numbers from the digits 0, 2, 3 and 5, the unit's position can be filled up in 2 ways and the ten's position can be filled up by 3 non-zero digits. Therefore, the number of 2-digit odd numbers $= 2 \times 3 = 6$

Further 1-digit odd numbers can be formed in 2 ways only.
So, the number of odd numbers that are less than 1000 can be formed is $24+6+2 = 32$.

Example 7 (a) How many arrangements can be made with the letters of the word 'MATHEMATICS'?
(b) In how many words, vowels occur together?
(c) In how many words, vowels occupy odd places?

Solution: (a) There are 11 letters in the word MATHEMATICS, where M occurs 2 times, A occurs 2 times, T occurs 2 times and rest of the letters are distinct.

Thus, the total number of arrangements is

$$\frac{11!}{2!\,2!\,2!} = 4989600$$

(b) The word MATHEMATICS contains 4 vowels. To find the total number of arrangements where vowels occur together, we take 4 vowels as one letter only. That is, we consider the word consisting of 8 letters out of which one is (AAEI). Out of these 8 letters, M occurs 2 times and T occurs 2 times; they can be arrangement in $\frac{8!}{2!\,2!}$ ways

Therefore, the total number of arrangements when vowels are taken together is $\frac{8!}{2!\,2!} \times \frac{4!}{2!} = 120960$

(c) Here we have to calculate the total number of arrangements so that vowels will occupy only odd places. There are 6 odd places out of 11 letters. 6 places can be filled up with 4 vowels A, A, E, I in $\frac{^6P_4}{2!}$ ways. The remaining 7 places can be filled up with 7 letters where M occurs 2 times and T occurs 2 times in $\frac{7!}{2!\,2!}$ ways.

Therefore, the total number of arrangements in which vowels occupy only the odd places is

$$\frac{^6P_4}{2!} \times \frac{7!}{2!\,2!} = 226800$$

Example 8 How many numbers not more than 5 digits can be formed with the digits 1, 2, 3, 4 and 5, with repetition being allowed?

Solution: There are five numbers 1, 2, 3, 4 and 5 and the repetition is allowed. Thus we can form 1-digit, 2-digit, 3-digit, 4-digit and 5-digit numbers.

So, Using these 5 digits, the number of 1-digit numbers $= 5^1 = 5$

Using these 5 digits, the number of 2-digit numbers is $= 5^2 = 25$

Using these 5 digits, the number of 3-digit numbers $= 5^3 = 125$

Using these 5 digits, the number of 4-digit numbers $= 5^4 = 625$

Using these 5 digits, the number of 5-digit numbers $= 5^5 = 3125$

Using these 5 digits, The number

Therefore, the total number of numbers not more than 5 digits can be formed with the digits 1, 2, 3, 4 and 5 is

$$5 + 25 + 125 + 625 + 3125 = 3905.$$