Hence, the present value of a future amount after n year can be written as

$$P = \frac{A_n}{\left(1+\frac{r}{100}\right)^n} = A_n\left(1+\frac{r}{100}\right)^{-n} = A_n(1+i)^{-n} \quad \cdots (2)$$

To compute the rate of interest, one can use the following formula:

$$\left(1+\frac{r}{100}\right)^n = \frac{A_n}{P}$$

or,
$$r = 100\left[\left(\frac{A_n}{P}\right)^{1/n} - 1\right] \quad \cdots (3)$$

Again, to find the ~~tex~~ time period, the following formula may be used:

$$\log A_n = \log P + n\log\left(1+\frac{r}{100}\right) = \log P + n\log(1+i)$$

or,
$$n = \frac{\log A_n - \log P}{\log\left(1+\frac{r}{100}\right)} = \frac{\log A_n - \log P}{\log(1+i)} \quad \cdots (4)$$

Finally, to find the compound interest after n time periods, one can use the formula given by the following:

$$I = A_n - P = P\left[\left(1+\frac{r}{100}\right)^n - 1\right] = P\left[(1+i)^n - 1\right] \quad \cdots (5)$$

If the interest is compounded k times in a year, then equation (1) becomes

$$A_n = P\left(1 + \frac{r/k}{100}\right)^{k \times n} = P\left(1 + \frac{i}{k}\right)^{k \times n} \quad \ldots (6)$$

That is, the amount after n years will be

$A_n = P\left(1 + \frac{r}{100}\right)^n = P(1+i)^n$, when the interest is compounded annually

$A_n = P\left(1 + \frac{r}{200}\right)^n = P(1+i/2)^n$, when the interest is compounded half-yearly

$A_n = P\left(1 + \frac{r}{400}\right)^n = P\left(1+\frac{i}{4}\right)^n$, when the interest is compounded quarterly.

$A_n = P\left(1 + \frac{r}{1200}\right)^n = P\left(1+\frac{i}{12}\right)$, when the interest is compounded annually

Interest compounded ~~annually~~ continuously.

If more frequently, we compute the interest, the more will be the compounded ~~interest~~ amount. If the k is very large, then the interest is said to compounded continuously. Hence, in case of continuous compounding, we have the amount after n years as

$$A_n = \lim_{k \to \infty} P\left[1 + \frac{r/k}{100}\right]^{k \times n}$$

$$= \lim_{k \to \infty} P\left[1 + (i/k)\right]^{k \times n}$$

$$= \lim_{k \to \infty} P\left[\left(1 + i/k\right)^{k/i}\right]^{i \times n}.$$

$$= \lim_{x \to \infty} P\left[\left(1+\frac{1}{x}\right)^x\right]^{n \times i} \quad \left[\text{Putting } \frac{x}{i} = x\right]$$

Again, we know that,

$$\lim_{x \to \infty}\left(1+\frac{1}{x}\right)^x = e$$

So, $A_n = Pe^{n \times i} = P.e^{n \times \frac{r}{100}}$

Amount at the changing rates of interest

For the principal P, if the rates of interest changing from time to time, i.e., at the rate of r_1% p.a (per annum) for the first year, at the rate of r_2% p.a for the second year, at the rate of r_3% p.a for the third year and so on. Finally, at the rate of r_n% p.a for the nth year, then the amount is given by

$$A_n = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)\cdots\left(1+\frac{r_n}{100}\right) \quad \cdots (8)$$

As a special case, the amount, at the rate of r_1% p.a, for the first n_1 years and at the rate of r_2% p.a for next n_2 years so that $n_1 + n_2 = n$, will be

$$A_n = P\left(1+\frac{r_1}{100}\right)^{n_1}\left(1+\frac{r_2}{100}\right)^{n_2} \quad \cdots (9)$$

Similarly, at the rate of $r_1\%$ p.a for first n_1 years, at the rate of $r_2\%$ p.a for next n_2 years and at the rate of $r_3\%$ p.a for the last n_3 years so that $n_1+n_2+n_3=n$, then the amount due after n years will be

$$A_n = \left(1+\frac{r_1}{100}\right)^{n_1}\left(1+\frac{r_2}{100}\right)^{n_2}\left(1+\frac{r_3}{100}\right)^{n_3} \quad \text{---(10)}$$

and so on.

Nominal and effective rate of interest

When the interest is calculated more than once a year, the declared annual rate of interest is known as nomial rate of interest. But the actual rate of interest compounded annually is known as the effective rate of interest.

If $r\%$ be the rate of interest on Rs 100 for one year, then r is called the nominal rate of interest per annum. Again if the interest is payable k times a year then the amount Rs 100 at the end of one