year will be

$$100\left[1+\frac{r/k}{100}\right]^{k} = P\left[1+(i/k)\right]^{k}$$

Hence, the interest after one year will be

$$100\left[1+\frac{r/k}{100}\right]^{k}-100 = 100\left[\left(1+\frac{r/k}{100}\right)^{k}-1\right]$$

Therefore, the effective rate of interest per annum will be $\left[\left(1+\left(\frac{r/k}{100}\right)\right)^{k}-1\right]$ or $\left[\left(1+\frac{i}{k}\right)^{k}-1\right]$

Which is, in general, greater than the nominal rate of interest per annum.

Growth and Depreciation

One may use the compound interest formula in the case where the value of money is uniformly increases at a constant rate. If the value increases at the rate of $r\%$ per annum, then after n years the final amount will be

$$A_n = P\left[1+\frac{r}{100}\right]^{n} = P\left[1+i\right]^{n}.$$

Again, if the value decreases at the rate of $r\%$ per annum, then after n years, the final amount will be $A_n = P\left[1-\frac{r}{100}\right]^{n} = P\left[1-i\right]^{n}.$

If the values decreases half-yearly at the rate of $r\%$ per annum, then after n years the final amount will be $A_n = P\left[1 - \frac{(r/2)}{100}\right]^{2n}$ and ~~also~~ ~~values~~ if the value decreases quarterly at the rate of r% per annum, then after n years the final amount will be $A_n = P\left[1 - \frac{r/4}{100}\right]^{4n}$ and so on.

Thus if the value of an asset or money decreases with respect to time, then it is a problem of depreciation.

<u>Example 1</u> Find the compound interest on Rs. 6250 at the rate of 14% per annum for 2 years compounded annually

<u>Solution:</u> Using the ~~sm~~ formula of compound interest, we get $I = A_n - P = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

Now, substituting P = Rs. 6250, r = 14 and n = 2 in the above formula, we have

$$\text{Compound interest} = A_2 - P = 6250\left[\left(1+\frac{14}{100}\right)^2 - 1\right]$$
$$= 6250\left[(1+0.14)^2 - 1\right]$$
$$= 6250\left[(1.14)^2 - 1\right]$$
$$= 1872.50$$

Thus the required compound interest is Rs. 1872.50

<u>Example 2</u> What sum will amount to Rs. 5525 at

the rate of 10% per annum compounded yearly for 13 years?

Solution: Here the formula for the amount is given by

$$A_n = P\left[1+\frac{r}{100}\right]^n$$

Now, substituting $r=10$, $n=13$ and $A_{13}=5525$ in the above formula, we have

$$P\left[1+\frac{10}{100}\right]^{13} = 5525$$

or, $$P(1.1)^{13} = 5525$$

Taking natural logarithm of both sides, we have

$$\log P = \log 5525 - 13\log 1.1$$
$$= 3.7423 - 13\times 0.0414$$
$$= 3.2041$$

Hence $$P = 10^{3.2041} \simeq 1600$$

Thus the required sum is Rs 1600

Example 3 The difference between the compound interest and the simple interest on a certain principal at the rate of 10% per annum for two years is Rs. 52. Find the principal

Solution The formula for compound interest is given by $I_c = A_n - P = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$ and the formula for simple interest is given by

$$I_s = \frac{P \times r \times n}{100}$$

Thus, substituting $r=10$ and $n=2$ in the above formula $P\left[\left(1+\frac{10}{100}\right)^2-1\right] - \left[\frac{P\times 10\times 2}{100}\right] = 52$

or, $P\left[(1.1)^2-1\right] - \left[P\times 0.10\times 2\right] = 52$

or, $P\left[(1.1)^2-1 - 0.10\times 2\right] = 52$

or, $P\times 0.01 = 52$

or, $P = 5200$

Thus, the required principal is Rs. 5200.

Example 4 The compound interest on Rs 30,000 at the rate of 7% per annum for a certain period is Rs. 4347. Find the time period.

Solution: Using the formula of compound interest, we get Compound interest $= A_n - P = P\left[\left(1+\frac{r}{100}\right)^n-1\right]$

Now substituting $P = 30{,}000$ and $r=7$ in the above formula, we have

$$30{,}000\left[\left(1+\frac{7}{100}\right)^n-1\right] = 4347$$

or, $30{,}000\left[(1.07)^n-1\right] = 4347$

or, $(1.07)^n = \frac{4347}{30{,}000}+1 = 1.449$

or, $n = \frac{\log 1.1449}{\log 1.07} = 2$

Thus, the required time period is 2 years.