Example 5 If the population of a town increases every year by 2% of the population at the beginning of that year, in how many years will the total increase of population be 40%.

Solution: Let us consider the formula $A_n = P\left[1+\frac{r}{100}\right]^n$,

where P = initial population, $r=2$ and $A_n = 1.40P$.

Thus $1.40P = P\left[1+\frac{2}{100}\right]^n$

or, $1.40 = \left[1+\frac{2}{100}\right]^n$

or, $1.40 = (1.02)^n$

or, $n \times \log(1.02) = \log 1.40$.

or, $n \times (0.0086) = 0.1461$

or, $n = 17$ (approx.)

Thus the required time is 17 years.

Example 6 Find the compound interest paid by a borrower on Rs. 7000 for 3 years, if the rates of interest for three years are 7%, 8% and 8.5% respectively.

Solution: For this problem, let us consider the formula for amount with changing rates of interest

$$A_n = P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$$

Here substituting $n=3$, $P=7000$, $r_1 = 7$, $r_2 = 8$ and $r_3 = 8.5$, We get $A_3 = 7000\left(1+\frac{7}{100}\right)\left(1+\frac{8}{100}\right)\left(1+\frac{8.5}{100}\right)$

$$= 7000(1.07)(1.08)(1.085)$$

$$= 8776$$

Therefore the compound interest paid by a borrower on Rs 7000 for 3 years = R 8776 − Rs 7000 = Rs. 1776.

Example 7 Find the present value of Rs 2000 due in 6 years if money is worth 5% compounded half yearly

Solution: The formula for present value of a future amount after n years is given by

$$P = \frac{A_n}{\left(1+\frac{r/2}{100}\right)^{2n}}$$

Now, substituting $r = 5$, $n = 6$ and $A_n = 2000$ in the above formula, we get

$$P = \frac{2000}{\left(1+\frac{5/2}{100}\right)^{12}} = \frac{2000}{(1+0.025)^{12}} = \frac{2000}{(1.025)^{12}}$$

or, $\log P = \log 2000 - 12 \log 1.025$

$$= 3.30103 - 12(0.01072)$$

$$= 3.1724$$

Hence $P = 1487.30$

Thus the required present value is Rs 1487.30

Example 8 A machine is depreciated at the rate of 10% on reducing balance. The original cost was Rs. 10,000 and the ultimate scrape value was Rs 3,750. Find the effective life of the machine.

Solution: To solve the above problem let us consider the formula for depreciation as

$$A_n = P\left[1 - \frac{r}{100}\right]^n$$

Substituting $P = 10{,}000$, $r = 10$ and $A_n = 3750$ in the above formula, we get

$$3750 = 10000\left[1 - \frac{10}{100}\right]^n$$

or, $3750 = 10000\,(0.9)^n$

or, $n \cdot \log 0.9 = \log 3750 - \log 10000$

or, $n = 9.3$

Thus, the effective life of the machine is 9.3 years.

Example 9 Find the effective rate of interest equivalent to nomial rate of 6% compounded quarterly.

Solution: The formula for the effective rate of interest is given by $\left[\left(1 + \frac{(r/k)}{100}\right)^k - 1\right]$

Here, $r = 6$ and $k = 4$

Thus, the effective rate of interest is

$$\left[\left(1+\frac{(6/4)}{100}\right)^4-1\right]$$

$$= \left[\left(1+\frac{6}{4\times100}\right)^4-1\right]$$

$$= \left[(1+0.015)^4-1\right]$$

$$= 1.0613-1$$

$$= 0.0613$$

Hence the effective of rate of interest is 6.13%.

Example 10 If interest is compounded continuously, at what annual rate will an amount be four times in 15 years?

Solution: Let r% be the annual rate of interest. Then by using the formula of continuous compounding, we have

$$A = Pe^{n\times i} = Pe^{n\times\frac{r}{100}}$$

Here $A = 4P$ and $n = 15$

Thus $4P = Pe^{15\times i}$

or, $i = \frac{\log 4}{15\log e} = \frac{0.6021}{15\times0.4343} = \frac{0.6021}{6.5145}$

$= 0.092$

Therefore, the annual rate of interest compounded continuously is 9.2%