Example 11 Find the present value of Rs. 10,000 due in 12 years at 6% per annum compound interest.

Solution: Here the present value of a future amount after n years can be written as

$$P = \frac{A_n}{\left(1+\frac{r}{100}\right)^n}$$

Now, substituting $r=6$, $n=12$ and $A_6 = 10,000$ in the above formula, we have

$$P = \frac{10000}{\left(1+\frac{6}{100}\right)^{12}} = \frac{10000}{(1.06)^{12}}$$

or, $\log P = \log 10000 - 12 \log 1.06 = 3.6964$

or, $P = 4971$.

Hence, the required present value is Rs. 4971.

Example 12 A machine depreciated 10% p.a. for the first two years and 7% p.a. for the next three years, depreciation being calculated on the diminishing value. If the value of the machine be Rs. 10,000 initially, find the average rate of depreciation and the depreciated value of the machine at the end of fifth year.

Solution: If the value of the machine decreases at the rate of r_1% p.a for the first n_1 years

and at the rate of r_2% p.a for the next n_2 years so that $n_1 + n_2 = n$, then after n years, the depreciated value of the machine will be

$$A_n = P\left(1 - \frac{r_1}{100}\right)^{n_1}\left(1 - \frac{r_2}{100}\right)^{n_2}$$

Now, substituting $P = 10,000$, $r_1 = 10$, $n_1 = 2$, $r_2 = 7$ and $n_2 = 3$, we get

$$A_5 = 10000\left(1 - \frac{10}{100}\right)^2\left(1 - \frac{7}{100}\right)^3$$

or, $$A_5 = 10000\,(0.9)^2(0.93)^3$$

$$= 6515.30$$

Hence, the depreciated value of the machine at the end of fifth year is Rs. 6515.30

Again, if r. is the average rate of depreciation, we can use the following formula for computation of r.

$$\left(1 - \frac{r}{100}\right)^n = \left(1 - \frac{r_1}{100}\right)^{n_1}\left(1 - \frac{r_2}{100}\right)^{n_2}$$

or, $$\left(1 - \frac{r}{100}\right)^n = (0.90)^2(0.93)^3$$

or, $$\left(1 - \frac{r}{100}\right)^5 = 0.65153$$

or, $$5\log\left(1 - \frac{r}{100}\right) = \log 0.65153$$

or, $$\left(1 - \frac{r}{100}\right) = 0.9179$$

or, $$\frac{r}{100} = 0.0821$$

$$r = 8.21$$

or,

Thus the average rate of depreciation is 8.21%.

Example 13 A man divided a sum of Rs. 18,750 between his two sons of age 10 and 13 years respectively in such a way that each would receive the same amount (at 3% p.a. compound interest) on attaining the age of 30 years. Find the original share of the younger son.

Solution Let Rs. x be the original share of the younger son. Then the original share of the elder son will be Rs. (18,750 − x). Now, it is given that the amount received by the elder son after 17 years is the same amount received by the younger son after 20 years.

Thus, $(18750-x)\left(1+\frac{3}{100}\right)^{17} = x\left(1+\frac{3}{100}\right)^{20}$

or, $(18750-x)(1.03)^{17} = x(1.03)^{20}$

or, $(18750-x) = x(1.03)^{3}$

or, $18750 = x(1+1.092727)$

or, $x = \frac{18750}{2.092727} = 8960 \text{ (approx)}$

Thus, the original share of the younger son is Rs. 8960.

Example 14 If interest compounded at the end of each year, find the compound interest on Rs. 10,000 at 5% p.a. for 2 years.

Solution: Using the formula of compound interest,

we get Compound interest $= A_n - P = P\left[\left(1+\frac{r}{100}\right)^n - 1\right]$

Now, substituting $P =$ Rs. 10000, $r = 5$ and $n = 2$ in the above formula, we have

$$10000\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

$$= 10000\left[(1.05)^2 - 1\right]$$

$$= 10000\,(0.1025)$$

$$= 1025$$

Thus, the required interest is Rs. 1025.

Exercises

1. A loan earns interest at a certain compound rate per annum. Four years ago the amount was Rs. 81, now it is Rs. 144. What will the amount be two years hence?
2. Find what time a sum of money will double itself at 4.5% interest compounded annually?
3. At what rate per annum compound interest will Rs. 100 amount to Rs. 500 in 12 years?
4. If the population of town increases every year by 1.5% of the population at the beginning of that year, in how many years will the total increase of the population be 40%.
5. A man deposits Rs. 5,000 in a savings bank