are $A(1+i)^{n-3}, A(1+i)^{n-4}, \ldots, A(1+i)$ and A.

So, the sum of this annuity is given by

$$S = A(1+i)^{n-1} + A(1+i)^{n-2} + \cdots + A(1+i) + A$$

$$= A\left[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + (1+i) + 1\right]$$

$$= A\left[1 + (1+i) + \cdots + (1+i)^{n-2} + (1+i)^{n-1}\right]$$

$$= A\,\frac{\left[(1+i)^{n} - 1\right]}{(1+i) - 1} \quad \left[\text{G.P series with common ratio } (1+i) > 1\right]$$

$$= \frac{A}{i}\left[(1+i)^{n} - 1\right]$$

Present value of immediate annuity or ordinary annuity.

The present value of an annuity represents the present values of all the payments. Let us consider an ordinary annuity of n payments of Rs. A each, where the interest rate per period is i and the first, second, ..., nth payments are due at the end ~~of~~ of one, two, ..., nth periods from the beginning.

The present value of the first payment $= A(1+i)^{-1}$,

The present value of the second payment $= A(1+i)^{-2}$

and so on. Finally, the present value of the nth payment $= A(1+i)^{-n}$.

Hence, the present value P of this annuity is given by

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \cdots + \frac{A}{(1+i)^n}$$

$$= \frac{A}{(1+i)^n}\left[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + 1\right]$$

$$= \frac{A}{(1+i)^n}\left[1 + (1+i) + \cdots + (1+i)^{n-1}\right]$$

$$= \frac{A}{(1+i)^n}\left[\frac{(1+i)^n - 1}{(1+i) - 1}\right] \quad \text{[G.P. series with common ratio } (1+i) > 1\text{]}$$

$$= \frac{A}{(1+i)^n}\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= \frac{A}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

$$= \frac{A}{i}\left[1 - (1+i)^{-n}\right]$$

Ammount of annuity due

An annuity due is an annuity where the payments are made at the beginning of each period. For example, a saving scheme of 10 years, in which equal payments are made at the beginning of each year, is an example of an annuity due. In this annuity of term 10 years, every payment is an investment, the first payment earns interest for 10 years, the second for 9 years and so on the last payment for 1 year. In general, the term of an annuity due begins at the time, ~~when the first payment earns interest for 10 years, the second for 9 years and so on the last payment~~ when the first payment is made and ends one period after last payment is made. The amount of an annuity due is the value of the annuity at the end of its term.

Let us consider an annuity of n payments of Rs. A each, with the interest rate per period is i and the first payment is due at the beginning of the term. Let S be the amount of this annuity due.

Thus, the first payment of Rs. A is made at the beginning of the first period, which earns interest for n periods, the second payment earns interest for $(n-1)$ periods and so on. Finally, the nth payment earns interest for one period.

Amount of the first payment $= A(1+i)^n$. Amount of the second payment $= A(1+i)^{n-1}$ and so on. Finally, the amount of the last payment $= A(1+i)$. Hence, S can be computed as.

$$S = A(1+i)^n + A(1+i)^{n-1} + \dots + A(1+i)$$

$$= A(1+i)\left[(1+i)^{n-1} + (1+i)^{n-2} + \dots + 1\right]$$

$$= (1+i)A\left[1 + (1+i) + (1+i)^2 + \dots + (1+i)^{n-1}\right]$$

$$= (1+i)A\left[\frac{(1+i)^n - 1}{(1+i)-1}\right]$$ [G.P. series with common ratio $(1+i) > 1$]

$$= (1+i)A\left[\frac{(1+i)^n-1}{i}\right]$$

$$= (1+i)\frac{A}{i}\left[(1+i)^n-1\right]$$

Present value of annuity due

The present value of annuity due is the sum of the present values of all the payments.

Let Rs. P be the present value of an annuity due of Rs. A per payment for n time periods at the rate of interest i per period. This is equivalent to a sum of Rs. P invested at an interest rate of i per time period, then a series of n equal payments of Rs. A each is obtained for n periods, the first payment being due at the time of initial investment.

Let us consider an annuity consisting of n payments of Rs. A each where the rate of interest is i per period and each payment is due at the beginning of payment period.

Then, The ~~present~~ present value of the first payment $= A$, the present value of the second payment $= A(1+i)^{-1}$ and so on. Finally, the present value of the nth payment $= A(1+i)^{-(n-1)}$

If P be the present value of this annuity due, then

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