Example 9 In how many of the permutations of 15 different things taken 4 at a time, one particular thing (a) never occurs, (b) always occurs?

Solution: (a) Let us keep aside the particular thing that will never occur. Thus $(15-1) = 14$ things are to be arranged taking 4 at a time is ${}^{14}P_4 = 14 \times 13 \times 12 \times 11 = 24024$

Therefore the required number of permutations is 24024.

(b) In this case, one particular thing always occurs. That is, 3 things are to be arranged from 14 things in ${}^{14}P_3$ ways. The particular thing can be placed in the first place or second place or Third place or a fourth place. Therefore the required number of permutations is $4 \times {}^{14}P_3 = 4 \times 14 \times 13 \times 12 = 8736$

Example 10 How many four-digits numbers can be formed with the numbers 1, 2, 3, 4 taken only one at a time? How many of them will ~~such~~ ~~would~~ be even numbers?

Solution: There are four distinct digits 1, 2, 3, 4 and the numbers can be taken only one at a time. Hence the number of 4-digit numbers that can be formed from 4 distinct numbers is ${}^4P_4 = 4 \times 3 \times 2 \times 1 = 24$

Further, in this case, a four digit number is even if the unit's place is occupied by 2 or 4. This can be done in 2 ways. Then the other 3 places (ten's, hundreds', and thousands') can be occupied by the rest 3 numbers. So, the number of permutations of 4 digits so that the

4-digit number formed is an even number is

$$2 \times {}^{3}P_{3} = 2 \times 3 \times 2 \times 1 = 12$$

Example 11 How many four-digit numbers can be formed with digits 1, 2, 5, 6, 7 no digit being repeated? How many of them are divisible by 5?

Solution There are 5 different digits 1, 2, 5, 6, 7 and repetition is not allowed. Thus the number of permutations of four digits taken from 5 digits 1, 2, 5, 6, 7 is ${}^{5}P_{4} = 5 \times 4 \times 3 \times 2$

$= 120$

Again, we need to calculate the number of 4-digit numbers that can be formed from these 5 digits which are divisible by 5. In this case, a number is divisible by 5 if 5 occupies the unit's place and other 3 places (ten's, hundred's and thousand's) are occupied by any 3 out of the remaining 4 numbers. Thus, the number of four-digit numbers that can be formed from 5 digits that are divisible by 5 is

$${}^{4}P_{3} = 4 \times 3 \times 2 = 24$$

Example 12 How many words that can be formed with the letters of the word 'STATISTICS'?

Solution: There are 10 letters in the word STATISTICS, out of which S occurs 3 times, T occurs 3 times, I occurs 2 times and the remaining 2 letters are distinct.

Thus the required number of words that can be formed with the letters of STATISTICS is

$$\frac{10!}{3!\,3!\,2!} = 50400$$

Example 13 In how many ways the letters of the word 'BALLOON' be arranged so that two L's do not come together?

Solution: There are seven letters in the word BALLOON with two L's and two O's. Total number of arrangements of the word is

$$\frac{7!}{2!\,2!} = \frac{7\times6\times5\times4\times3\times2\times1}{4} = 1260$$

To find the number of arrangements of the word BALLOON so that two L's do not come together, we proceed as follows:

First of all, we find the number of arrangements where two L's come together. That is, if two L's are taken together and treated as single letter then there will be only 6 letters with two O's. Hence, the number of arrangements of 6 letters with two O's and rest four are distinct is

$$\frac{6!}{2!} = \frac{6\times5\times4\times3\times2\times1}{2} = 360$$

Therefore, the number of arrangements in which two L's do not come together is $1260 - 360 = 900$.

Exercises

1. Find the value of (i) ${}^{7}P_{3}$ (ii) ${}^{6}P_{6}$ (iii) ${}^{10}P_{2}$ (iv) ${}^{13}P_{1}$.
2. What is the value of n when ${}^{n}P_{2} = 12$?
3. Find the value of n when ${}^{n}P_{3} = 6 \cdot {}^{n}P_{2}$.
4. Prove that ${}^{n}P_{r} = {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$
5. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_{n} = 3 : 5$, then find the value of n.
6. In how many different ways, can the letters of the word 'SALOON' be arranged so that two Os do not come together?
7. In how many ways, can the letters of the word 'ASSISTANT' taken all together be arranged?
8. How many number between 100 and 1000 can be formed with the digits 2, 3, 4, 0, 8, 9 each digit occuring only once in number?
9. How many odd number of 5 digit can be formed with the digits 1, 2, 3, 4, 5, 6 without repetitions?
10. How many words can be formed with the letters of the word 'MONDAY'? How many of these words begin with M and do not end with Y?
11. How many different permutations can be made with the letters of the word 'CONSTANT' so that the vowels are always together?