Solution: Here we have to select 5 persons from 7 boys and 8 girls, which will include exactly 2 boys. So, we must have $(5-2) = 3$ girls in the selection. Now, we have to select 2 boys from 7 in ${}^{7}C_{2}$ ways and 3 girls from 8 in ${}^{8}C_{3}$ ways, therefore the total number of selections

$$= {}^{7}C_{2} \times {}^{8}C_{3} = \frac{7\times6}{2!} \times \frac{8\times7\times6}{3!}$$

$$= 1176$$

Example 6 Out of 12 electric bulbs, 4 are defectives. In how many ways, 5 bulbs can be selected, which includes at least one defective bulb

Solution: It is given that total number of bulbs is 12, out of which 4 are defectives and rest 8 are non-defectives. If the selection of 5 bulbs include at least one defective bulb, then we have the following cases:

Case	Defective	Non-defective
I	1	4
II	2	3
III	3	2
IV	4	1

In case I, 1 defective out from 4 bulbs 4 non-defectives from 8 bulbs can be selected in ${}^{4}C_{1} \times {}^{8}C_{4}$ ways. Similarly, in case II, bulbs can be selected in ${}^{4}C_{2} \times {}^{8}C_{3}$ way, in case III, in ${}^{4}C_{3} \times {}^{8}C_{2}$ ways and in case IV in

${}^{4}C_{4} \times {}^{8}C_{1}$ ways

So, the total number of selection, which will include at least one defective bulb is given by

$$= {}^4C_1 \times {}^8C_4 + {}^4C_2 \times {}^8C_3 + {}^4C_3 \times {}^8C_2 + {}^4C_4 \times {}^8C_1$$

$$= 4 \times 70 + 6 \times 56 + 4 \times 28 + 1 \times 8 = 736$$

Example 7 In an examination, a minimum mark is to be secured in each of the 8 subjects for a pass. In how many ways, can a candidate fail?

Solution: A candidate fails in the examination if he can not secure the minimum marks in one or more subjects. one subject can be selected out of 8 in ${}^8C_1$ ways, two subjects can be selected out of 8 in ${}^8C_2$ ways and so on. Finally, eight subjects can be selected out of 8 in ${}^8C_8$ ways.

Therefore the total number of ways a candidate can fail in the examination $= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$$= 2^8 - 1 = 255$$

Example 8 Six parallel straight lines in a plane are intersected by a set of 7 parallel straight lines. How many parallelograms are there in the network thus formed?

Solution: A parallelogram is formed by any two straight lines selected from the set of 6 parallel straight lines and any two straight lines selected from another set of 7 parallel straight lines. The two straight lines can be selected from a set of 6

parallel straight lines in ${}^6C_2$ ways, Again another 2 straight lines can be selected from a set of 7 parallel lines in ${}^7C_2$ ways. Thus the total number of parallelogram formed $= {}^6C_2 \times {}^7C_2 = 15 \times 21 = 315$

Example 9 A box contains 6 black balls, 7 white balls and 9 red balls. 3 balls are selected one by one without replacement. Find the number of ways of such selection such that

(a) all 3 balls are of different colours.

(b) all 3 balls are not of different colours.

Solution: The box contains 6 black balls, 7 white balls and 9 red balls. The total number of balls is $6+7+9 = 22$

(a) Here the balls are selected in such a way that all the balls are of different colours. This can occur in 6 different ordered as BWR or BRW or WRB or WBR or RBW or RWB. In each of such cases, the number of ways of such selection $= {}^6C_1 \times {}^7C_1 \times {}^9C_1 = 378$

Therefore, the required number of selections such that the balls are of different colours $= 3! \times 378 = 6 \times 378 = 2268$

(b) Here first of all we compute the total number of ways of selecting 3 balls from the box. Since the total number of ball is 22, so the first ball can be drawn in ${}^{22}C_1$ ways. Again, the balls are drawn

one by one without replacement, so the second ball can be drawn in $^{21}C_1$ ways. Similarly, the third ball can be drawn in $^{20}C_1$ ways. Therefore, the total number of ways of selecting 3 balls one by one without replacement

$$= {}^{22}C_1 \times {}^{21}C_1 \times {}^{20}C_1 = 9240$$

Hence, the total number of selections such that all three are not of different colours $= 9240 - 2268 = 6972$

Example 10 In a question paper, there are two groups each containing 6 questions. A candidate is required to answer 7 questions taking at least 3 from each group. In how many ways, 7 questions can be selected?

Solution: There are two groups of questions in a question paper, say group I and group II. The candidate can select questions in the following way:

4 questions from group I and 3 questions from group II and 3 questions from group I and 4 questions from group II

Thus, in the first case, there are ${}^6C_4 \times {}^6C_3$ ways and in the second case, there are ${}^6C_3 \times {}^6C_4$ ways. Hence the total number of selections $= 2 \times {}^6C_3 \times {}^6C_4 = 2 \times 20 \times 15 = 600$

Example 11 How many words (each word containing 2 vowels and 3 consonants) can be formed with the letters of the word FACETIOUS?