Solution: There are 9 different letters in the word FACETIOUS of which 5 are vowels and 4 are consonants. We can select 2 vowels out of 5 in ${}^5C_2$ ways and 3 consonants out of 4 in ${}^4C_3$ ways. Therefore, 2 vowels and 3 consonants can be selected from 9 letters in ${}^5C_2 \times {}^4C_3$ ways. In each of the ${}^5C_2 \times {}^4C_3$ selections, there are 5 distinct letters (2 vowels and 3 consonants) that will form different words by arranging them in 5! ways.

Hence the total number of words $= {}^5C_2 \times {}^4C_3 \times 5!$

$= 10 \times 4 \times 120$

$= 4800$

Example 12 Find the number of ways in which 9 boys can be divided into 3 groups, each group containing 3 members.

Solution: The number of ways in which 9 ~~boys~~ boys can be divided into 3 groups, each group containing 3 ~~elements~~ members $= \dfrac{9!}{3!\,(3!)^3} = \dfrac{9\times8\times7\times6\times5\times4}{6\times6\times6} = 280$

Example 13 There are 3 sections in a question paper, each containing 4 questions. A candidate has to answer any 8 questions choosing at least 2 questions from each section. In how many ways, the candidate can answer the questions?

Solution: Suppose 3 sections are A, B and C each containing 4 questions. 8 questions with at least 2 questions from each group can be selected in the following ~~manner~~ manner:

Case	Section A	Section B	Section C
I	2	3	3
II	2	2	4
III	3	3	2
IV	4	2	2
V	3	2	3
VI	2	4	2

In case I, the number of ways $= {}^4C_2 \times {}^4C_3 \times {}^4C_3 = 6 \times 4 \times 4 = 96$

In case II, the number of ways $= {}^4C_2 \times {}^4C_2 \times {}^4C_4 = 6 \times 6 \times 1 = 36$

In case III, the number of ways $= {}^4C_3 \times {}^4C_3 \times {}^4C_2 = 4 \times 4 \times 6 = 96$

In case IV, the number of ways $= {}^4C_4 \times {}^4C_2 \times {}^4C_2 = 1 \times 6 \times 6 = 36$

In case V, the number of ways $= {}^4C_3 \times {}^4C_2 \times {}^4C_3 = 4 \times 6 \times 4 = 96$

In case VI, the number of ways $= {}^4C_2 \times {}^4C_4 \times {}^4C_2 = 6 \times 1 \times 6 = 36$

Hence, the total number of ways the candidate can answer the question $= 96 + 36 + 96 + 36 + 96 + 36 = 396$

Example 14 From 10 books, in how many ways, can a selection of 6 books be made so that two specified books are always (a) included and (b) excluded?

Solution: (a) There are 10 books. We have to select 6 books such that 2 specified books are always included. That is, we have to select $(6-2)=4$ books from the remaining $(10-2)=8$ books. This can be done in ${}^8C_4$ ways

Then, the required number of selections

$$= {}^8C_4 = \frac{8!}{4!\,4!} = \frac{8\times7\times6\times5}{4\times3\times2\times1} = 70$$

(b) Here 2 specified books of 10 are always excluded. So, we have to select 6 books from 10 books excluding these particular 2 books. Thus, 6 books are to be selected from remaining $(10-2)=8$ books in ${}^8C_6$ ways.

So, the required number of selections

$$= {}^8C_6 = {}^8C_2 = \frac{8\times7}{2\times1} = 28$$

Example 15 A ~~committee~~ Committee of 5 persons is to be formed out of 6 gentlemen and 4 ladies. In how many ways, the committee can be formed if

(a) a particular lady must be in the committee?

(b) two particular gentlemen refuse to work in the same Committee?

(c) The Committee include at least one lady?

Solution: (a) In this case, we have to form a Committee of 5 persons from 6 gentlemen and 4 ladies where a particular lady must be in the committee.

So, we have to select the rest 4 persons for the committee from 6 gentleman and 3 ladies, which can be selected in $^9C_4$ ways. Then the required number of ways $= {^9C_4} = \frac{9!}{4!\,5!} = \frac{9\times8\times7\times6}{4\times3\times2\times1} = 126$

(b) Here, first of all, we consider 2 particular gentlemen in a committee who refuses to work in the same committee. So, we have to select rest 3 persons for the committee from 4 gentlemen and 4 ladies, which can be done in $^8C_3$ ways. Again the total number of ways a committee can be formed without any restrictions is in $^{10}C_5$ ways.

Therefore the required number of ways

$$= {^{10}C_5} - {^8C_3} = \frac{10!}{5!\,5!} - \frac{8!}{3!\,5!} = 252 - 56 = 196$$

(c) The number of ways the committee can be selected when there is no lady is $^6C_5$. Therefore, the number of ways such that the committee will include at least one lady

$$= {^{10}C_5} - {^6C_5} = \frac{10!}{5!\,5!} - \frac{6!}{5!\,1!} = 252 - 6 = 246$$

<u>Example 16</u> If $^nC_2 = 45$, find n

Solution: It is given that $^nC_2 = 45$

$$\text{or} \quad \frac{n!}{2!\,(n-2)!} = 45$$