$$\text{Or,} \quad \frac{n(n-1)}{2} = 45$$

$$\text{or,} \quad n(n-1) = 90$$

$$\text{or,} \quad n^2 - n - 90 = 0$$

$$\text{Or,} \quad n^2 - 10 + 9n - 90 = 0$$

$$\text{or,} \quad n(n-10) + 9(n-10) = 0$$

$$\text{Or,} \quad (n-10)(n+9) = 0$$

So, $n = 10$ or -9

Since, n can not be a negative integer, $n = 10$

Example 17 If ${}^{n}C_7 = {}^{n}C_{11}$, prove that ${}^{21}C_n = 1330$

Solution: we are given that ${}^{n}C_7 = {}^{n}C_{11}$

Also we know that ${}^{n}C_r = {}^{n}C_{n-r}$

Thus ${}^{n}C_7 = {}^{n}C_{n-7} = {}^{n}C_{11}$

Hence, we get $n - 7 = 11$

$$\text{or,} \quad n = 11 + 7 = 18$$

Now ${}^{21}C_n = {}^{21}C_{18} = {}^{21}C_3 = \frac{21!}{3!18!} = \frac{21 \times 20 \times 19}{3 \times 21} = 1330$

Example 18 If ${}^{18}C_r = {}^{18}C_{r+2}$, find ${}^{r}C_5$

Solution: we have, ${}^{18}C_r = {}^{18}C_{r+2}$

$$\text{or,} \quad {}^{18}C_r = {}^{18}C_{18-r-2} \quad \left[\because {}^{n}C_r = {}^{n}C_{n-r}\right]$$

or, $r = 18 - r - 2$ or, $2r = 16$, or, $r = 8$

Now $^{r}c_5 = {}^{8}c_5 = {}^{8}c_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

Exercises

1. If $^{n}c_7 = {}^{n}c_3$, find the value of n

2. If $^{20}c_r = {}^{20}c_{2r+5}$, then find the value of r.

3. Find n and r if

$$^{n}c_{r-1} : {}^{n}c_r : {}^{n}c_{r+1} = 2:3:4$$

4. Prove that $^{n}c_r + {}^{n-1}c_{r-1} + {}^{n-1}c_{r-2} = {}^{n+1}c_r$

5. If $^{22}c_r = {}^{22}c_{r+1}$, find the value of $^{r}c_4$

6. In how many ways a committee of 5 be formed from 4 professors and 6 students so as to include at least 2 professors?

7. From 6 bowlers, 2 wicket keepers and 8 batsmen, in how many ways a team of 11 players consisting of at least 4 bowlers, one wicketkeeper and at least 5 batsmen be formed?

8. In an examination paper, there 7 questions in group A, out of which any 4 are to be attempted by a

Candidate and there are 6 questions in group B, out of which any 3 are to be attempted. In how many different ways can the paper be attempted in full?

9. From a class of 12 boys and 10 girls, 10 students are to be chosen for a competition, including at least 4 boys and 4 girls. The 2 girls who won the prize last year must be included. Find the number of ways of such a selection.

10. There are 9 consonants and 5 vowels. How many words of 7 letters can be formed using 4 consonants and 3 vowels?

11. A football team of 11 players is to be selected from 2 groups consisting of 8 and 7 players. In how many ways, can the selection be made so that no groups contributes less than 5 players?

12. A box contains 7 red, 6 white and 4 black balls. How many selections of 3 balls can be made so that (a) all three are red balls, (b) none is red ball and (c) there is one ball of each colour?

13. In how many different ways, can 9 men be selected from 15 men so as to always

(a) include 3 particular men and (b) exclude 3 particular men?

14. There are 6 bowlers in a cricket team of 11 players. In how many ways, can a ~~team~~ team of 11 cricketers be chosen such that at least 4 bowlers are to be included?

BINOMIAL THEOREM

3.1 Introduction

Any expression involving only two terms is known as a binomial expression. For instance, $5+x$, $x+2y^2$, x^2-3, x^3-5y^2, etc. are some binomial expressions. A formula by which any binomial expression raised to a certain power can be expressed as a series is known as binomial theorem. The formula for the expression $(a+x)^2$ can be obtained directly by multiplying; ~~$(a+x)^3$~~ the expression as of $(a+x)^3$ can be also be obtained directly by multiplying. But it is very difficult to find $(a+x)^n$ by multiplying when n is very large positive integer. We use mathematical induction to prove a formula for $(a+x)^n$ for any integer n.

Now by direct multiplication,

$$(a+x)^1 = a+x = {}^1C_0\, a^1x^0 + {}^1C_1\, a^0x^1$$

$$(a+x)^2 = (a+x)(a+x) = a^2+2ax+x^2 = {}^2C_0\, a^2x^0 + {}^2C_1\, a^1x^1 + {}^2C_2\, a^0x^2$$

$$(a+x)^3 = (a+x)^2(a+x) = (a^2+2ax+x^2)(a+x) = a^3+3a^2x+3ax+x^3$$

$$= {}^3C_0\, a^3x^0 + {}^3C_1\, a^2x^1 + {}^3C_2\, a^1x^2 + {}^3C_3\, a^0x^3$$