Notes of my portion of syllabus of CC-1. I am Subhasundar Bandyopadhyay (SB in short)

Books : 1. For Calculus : (a) Differential Calculus - R.K. Ghosh and K.C Maity
(b) Calculus - Volume I & II - T. Apostol.

2. For Geometry : (a) Advanced Analytical Geometry - J.G. Chakraborty and P.R. Ghosh
(b) Analytical Geometry of two and three dimensions - R.M. Khan
(c) Co-ordinate Geometry - S.L. Loney
(d) Co-ordinate Geometry of Three dimensions - J.T. Bell.

Unit 1 Calculus :

1. Hyperbolic Functions : We define hyperbolic sine of x (written as $\sinh x$) $= \frac{1}{2}(e^{x} - e^{-x})$ hyperbolic cosine of x (written as $\cosh x$) $= \frac{1}{2}(e^{x} + e^{-x})$. Similarly, we define

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$; $\coth x = \frac{1}{\tanh x} = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$, $\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^{x} + e^{-x}}$

$\operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}}$. The following two results follow from our definitions :

$\cosh x + \sinh x = e^{x}$ and $\cosh x - \sinh x = e^{-x}$ and hence we have a formula $\cosh^2 x - \sinh^2 x = 1 \quad \ldots \quad (1)$

It also follows, $\operatorname{sech}^2 x = 1 - \tanh^2 x$ (dividing by $\cosh^2 x$ in (1))

$\operatorname{cosech}^2 x = \coth^2 x - 1$ (dividing by $\sinh^2 x$ in (1))

Addition Theorems :

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y \quad \ldots (2)$$

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y \quad \ldots (3)$$

$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y} \quad \ldots (4)$$

We prove $\cosh(x+h) = \cosh x \cosh y + \sinh x \sinh y$

Proof : $\cosh(x+y) = \frac{1}{2}\left[e^{(x+y)} + e^{-(x+y)}\right] = \frac{1}{2}\left(e^{x} \cdot e^{y} + e^{-x} \cdot e^{-y}\right)$

$= \frac{1}{2}\left\{(\cosh x + \sinh x)(\cosh y + \sinh y) + (\cosh x - \sinh x)(\cosh y - \sinh y)\right\}$

$= \cosh x \cosh y + \sinh x \sinh y$. Similarly, other results can be proved.

From addition theorem, putting $y=x$, we get

$$\cosh 2x = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x$$

$$\sinh 2x = 2\sinh x \cosh x \quad \text{and} \quad \tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$$

Properties of hyperbolic functions:

Hyperbolic Sine:

1. $\sinh(-x) = -\sinh x$; $\sinh 0 = 0$
2. $y = \sinh x$ is defined for every real value of x in $-\infty < x < \infty$ and the function is continuous.
3. $y = \sinh x$ steadily increases as x increases from $-\infty$ to ∞.

The graph of $y = \sinh x$ is shown in Figure 1

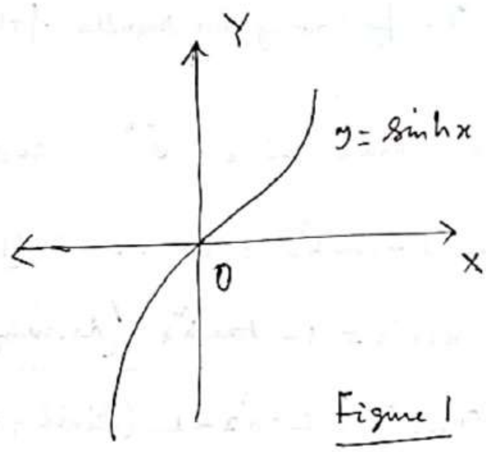


Figure 1

Hyperbolic cosine:

1. $\cosh(-x) = \cosh x$; $\cosh 0 = 1$
2. $y = \cosh x$ is defined for every value of x in $-\infty < x < \infty$ and is continuous. It is always positive as e^x and e^{-x} are positive.
3. $y = \cosh x$ decreases as x increases from $-\infty$ to 0 and then increases from 0 to ∞.
4. $\cosh x > \sinh x$ for all x (Since $\cosh x - \sinh x = 1/e^x > 0$)
5. The least value $(=1)$ of $\cosh x$ occurs when $x = 0$

6. The graph of $y = \cosh x$ is given in Figure 2. This curve is called a Catenary (it is the curve in which a uniform flexible chain with fixed ends hangs.)

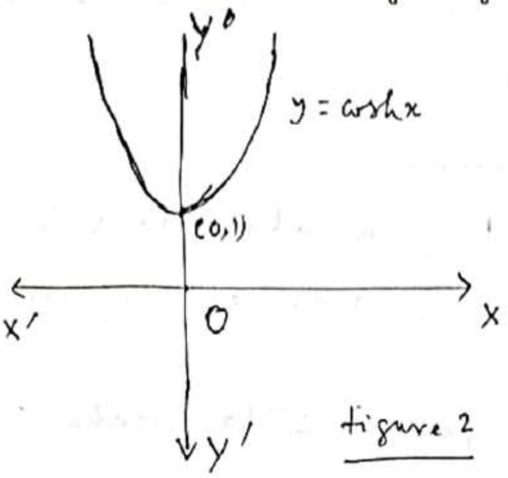


figure 2

Hyperbolic tangent and cotangent :

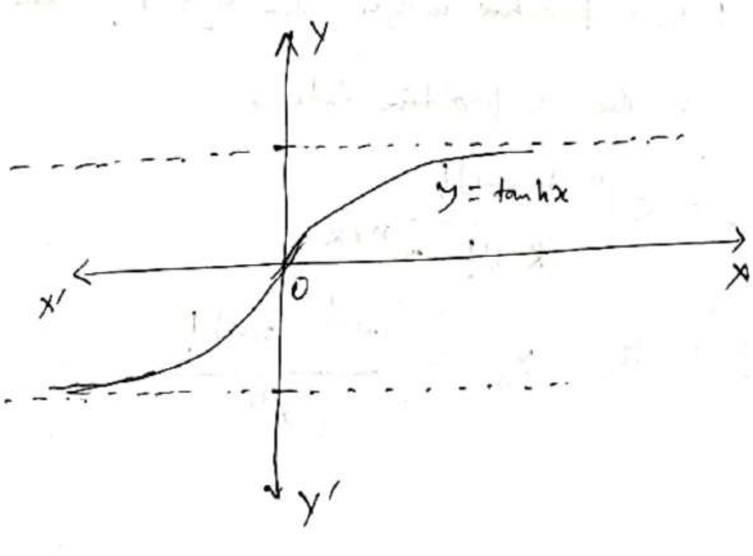
1. tanh x and coth x are both odd functions of x.

2. $\tanh 0 = 0$

3. $y = \tanh x$ is defined for every real x by coth y is defined for every real x except 0. It is an increasing function.

4. As x increases from 0 to ∞, tanh x increases from 0 to 1. As x increases from $-\infty$ to 0, tanh x increases from -1 to 0. $y = \pm$ are the asymptotes. So, $|\tanh x| < 1$, for all x. It then follows $|\coth x| > 1$.

In figure 3, graph of $y = \tanh x$ is shown



Hyperbolic secant and cosecant :

1. $0 < \text{sech}\,x \le 1$, Sech x has its maximum value 1 at $x = 0$,

$\text{sech}(-x) = \text{sech}\,x$

2. $\text{Cosech}(-x) = -\text{cosech}\,x$

3. When x grows large in absolute values ~~coth~~ ~~coth~~ cosech ~~x~~ approaches zero, sech also approaches zero.

Exercise 1. Draw the graphs of $\coth x$, $\text{sech}\,x$, and $\text{cosech}\,x$.

2. Prove that $1 - \cosh x = -2\sinh^2(x/2)$

3. Prove that $\cosh x - \cosh y = 2\sinh\frac{x+y}{2}\sinh\frac{x-y}{2}$

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2. Higher order derivatives

If $y = f(x)$, then we denote $\frac{d^n y}{dx^n} = f^n(x) = y_n$, when nth order derivative exists

Now we list the nth derivatives of some functions, where it exists. The proof can be done by using mathematical induction.

1. $y = x^k$, k is any real number.

$y_n = k(k-1)(k-2)\cdots(k-n+1)\,x^{k-n}$ $\forall$ positive integer n

Special cases : (a) If k be a positive integer, then $y_k = k!$ and $y_n = 0$ if $n > k$

(b) If $y = x^{-k}$, k be a positive integer

then $y_n = (-1)^n \frac{(k+n-1)!}{(k-1)!\, x^{n+k}}$

2. If $y = \log x$ $(x>0)$ then $y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$