Unit 1 Coordinate Geometry

1.1 Transformations of Rectangular axes

Equation curves in two dimension have reference to certain set of axes. In fact, we shall have different equation for the same curves when referred to different set of ~~axes~~ coordinate axes. This may happen when the origin is shifted to a point keeping the direction of axes the same or when the axes are rotated through the same angle keeping the origin unaltered. The former is called translation and the latter is called rotation. Change of coordinates may also be effected by a combination of the two, in either order, and is called a rigid motion. These transformations are also known as orthogonal transformations.

1.2 Change of origin without changing the direction of the axes (Translation)

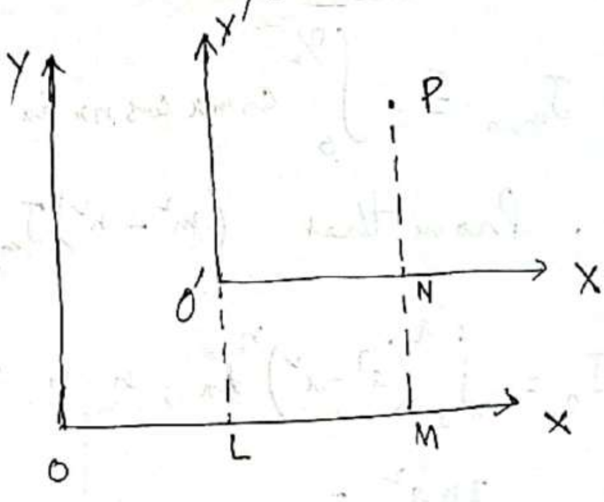

Figure-1

Let OX and OY be the set of rectangular axes referred to which the coordinates of an arbitrary point P be (x, y),

O is the origin. Let O', the new origin, be at (h, k) and $O'X'$, $O'Y'$ be the new set of axes parallel to the original axes. Let the coordinates P referred to the new set of axes be (x', y').

Then $x = OM = OL + LM = OL + O'N = h + x' = x' + h$

$y = PM = MN + NP = O'L + PN = k + y' = y' + k$

Hence, $x = x' + h$, $y = y' + k$ are the transformation formulae for the translation of axes to the new origin (h, k).

Then the equation $f(x, y) = 0$ with reference to the old set of axes becomes $f(x' + h, y' + k) = 0$ with reference to the new set of axes. Removing the primes to put it in general form, the new equation becomes $f(x + h, y + k) = 0$

Note: The above formulae can also be written as

$x' = x - h$, $y' = y - k$

1.3 Transformation from one pair of rectangular axes to another with the same origin (Rotation)

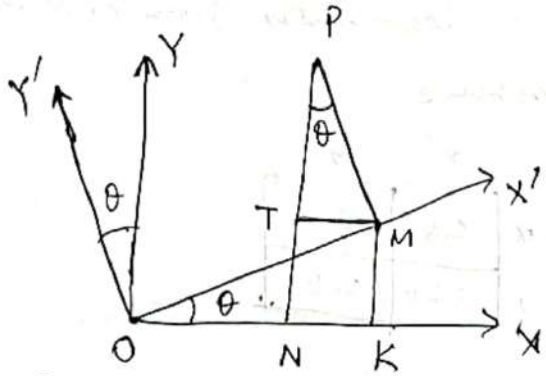

Figure 2

Let the axes OX and OY be turned about O through an angle θ to the position OX' and OY'. Let P be any point (x, y) referred to the system OX, OY and (x', y') referred to new set of axes OX', OY'.

PN and PM are drawn perpendiculars to OX and OX' respectively. Draw MK and MT perpendiculars to OX and PN respectively.

We have $x' = OM$, $y' = PM$

Then $x = ON = OK - NK = OK - TM = x'\cos\theta - y'\sin\theta$

Since $\angle TPM = 90° - \angle TMP = \angle TMO = \theta$.

Again $y = PN = TN + PT = MK + PT = x'\sin\theta + y'\cos\theta$

Hence $x = x'\cos\theta - y'\sin\theta \quad \ldots (1)$

and $y = x'\sin\theta + y'\cos\theta \quad \ldots (2)$

are the transformation formulae for rotation of axes.

Solving x' and y' from (1) and (2), we get

$$x' = x\cos\theta + y\sin\theta \quad \ldots (3)$$

$$y' = -x\sin\theta + y\cos\theta \quad \ldots (4)$$

The transformation of coordinates given by (1), (2) and (3), (4) are expressed by the scheme

	x'	y'
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

1.4 Translation followed by a rotation (or Rotation followed by a translation)

When the origin is shifted to the point (h, k), the coordinates of any point $P(x, y)$ becomes $P(x+h, y+k)$. Then the axes are turned through an angle θ. The coordinates of P referred to the new set of axes are obtained by substituting $(x\cos\theta + y\sin\theta)$, for x and $(x\sin\theta + y\cos\theta)$ for y. Hence the coordinates of P due to both translation and rotation (or rigid motion) P becomes $(x\cos\theta - y\sin\theta + h,\ x\sin\theta + y\cos\theta + k)$

As is obvious, the result will be same if rotation be followed by translation. So, we will write this case as translation and rotation,

Note : Writing $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$, $A = \begin{bmatrix} h \\ k \end{bmatrix}$,

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $S = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$,

the formulae for coordinate transformation ~~dest~~ due to (i) translation (ii) rotation and (iii) translation and rotation can be written in matrix notation as

(i) $X = IX' + A$, (ii) $X = SX'$ and (iii) $X = SX' + A$

respectively. From these formulae the inverse relations are