the coordinates of it in the old system

Solution: Let (x, y) be the coordinates of the point in the old system. By the formulae for translation and rotation

$$x = \frac{3-4\sqrt{3}}{2}\cos 60 - \frac{4+3\sqrt{3}}{2}\sin 60^\circ + 2$$

and $$y = \frac{3-4\sqrt{3}}{2}\sin 60 + \frac{4+3\sqrt{3}}{2}\cos 60^\circ + 1$$

So, $$x = \frac{3-4\sqrt{3}}{2} \times \frac{1}{2} - \frac{4+3\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 2$$

$$= \frac{3-4\sqrt{3}}{4} - \frac{4\sqrt{3}+9}{4} + 2$$

$$= \frac{3-4\sqrt{3}-4\sqrt{3}+9+8}{4}$$

$$= \frac{20-8\sqrt{3}}{4} = 5-4\sqrt{3}$$

$$y = \frac{3-4\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{4+3\sqrt{3}}{2} \times \frac{1}{2} + 1$$

$$= \frac{3\sqrt{3}-12}{4} + \frac{4+3\sqrt{3}}{4} + 1$$

$$= \frac{6\sqrt{3}-8+4}{4} = \frac{6\sqrt{3}-4}{4} = \frac{3\sqrt{3}-2}{2}$$

So, $x = 5-4\sqrt{3}$, $y = \frac{3\sqrt{3}-2}{2}$

3. If $ax+by$ transforms to $a'x'+b'y'$ under rotation of axes then show that $a^2+b^2 = a'^2+b'^2$

If $A = [a \ b]$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

then $ax+by = AX$. Let us consider the rotation given by $X = SX'$ where

$$S = \begin{bmatrix} \lambda & -\mu \\ \mu & \lambda \end{bmatrix}, \quad \lambda^2+\mu^2=1$$

Now $$AX = ASX' = \begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} \lambda & -\mu \\ \mu & \lambda \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$= (a\lambda+b\mu)x' + (-a\mu+b\lambda)y' = a'x'+b'y'$$

So, $a' = (a\lambda+b\mu)$ and $b' = -a\mu+b\lambda$

So, $$a'^2+b'^2 = (a\lambda+b\mu)^2+(-a\mu+b\lambda)^2$$
$$= (\lambda^2+\mu^2)(a^2+b^2) = a^2+b^2 \quad (\text{as } \lambda^2+\mu^2=1)$$

1.6 General equation of second degree in x and y

The general equation of second degree in x and y is usually written in the form

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \cdots (1)$$

The curve represented by (1) is called second order curve. Let us consider the notation

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc+2fgh-af^2-bg^2-ch^2$$

and $$D = \begin{vmatrix} a & h \\ h & b \end{vmatrix} = ab-h^2$$

Δ is called the discriminant of (1) and is invariant under translation and rotation of axes.

D is also invariant under translation and rotation of axes..

We can show that

if $\Delta = 0$, then the equation (1) represents a pair of straight lines. These two lines are intersecting if $D \neq 0$ and parallel (or coincident) if $D = 0$

If $\Delta = 0$, then the equation (1) is said to represent a degenerate conic.

If $a = b$ and $h = 0$, the equation (1) represents a circle.

If $\Delta \neq 0$, then the equation (1) represents either an ellipse or a hyperbola or a parabola (Non-degenerate conic). These are proper conics.

If $\Delta \neq 0$ and $D > 0$, (1) represents an ellipse (or circle)

If $\Delta \neq 0$, and $D < 0$, (1) represents a hyperbola

If $\Delta \neq 0$ and $D = 0$, (1) represents a parabola.

Central conic (Definition): If any chord of a conic through a particular point is bisected by the point then the conic is said to be central conic and that particular point is called the centre of the conic.

It can be shown that if the conic (1) is central and (α, β) is its centre, Then

$$a\alpha + h\beta + g = 0 \quad \ldots (2)$$

$$\text{and} \quad h\alpha + b\beta + f = 0 \quad \ldots (3)$$

From (2) and (3), we have

$$\alpha = \frac{fh - bg}{ab - h^2}, \quad \beta = \frac{gh - af}{ab - h^2}$$

So, a conic is a central conic if $ab - h^2 \neq 0$

So, ellipse or hyperbola are central conics and parabola is a non-central conic.

Canonical form: The equation (1), that is, a general equation of second degree in x and y can be reduced to the standard equation of a conic by suitable transformation of coordinates. The standard equation is also