values, the given equation reduces to

$X^2 - Y^2 = 0$ which represents the

Canonical form and represent two intersecting

straight lines $X + Y = 0$ and $X - Y = 0$

4. Reduce the following equation to Canonical form:

$$4x^2 - 4xy + y^2 + 2x - 26y + 9 = 0$$

Solution: Here $a = 4$, $b = 1$, $h = -2$, $g = 1$, $f = -13$, $c = 9$

So, $\Delta = -62 \neq 0$ and

$D = 0$

Hence the given equation represents a parabola

The given equation may be written as

$$(2x-y)^2 + 2x - 26y + 9 = 0$$

or, $(2x - y + \lambda)^2 = -2x + 26y - 9 + \lambda^2 + 4\lambda x - 2\lambda y$

$= 2(2\lambda - 1)x + 2(13 - \lambda)y + \lambda^2 - 9$, λ

being a Constant.

λ is so chosen that the two straight lines

$2x - y + \lambda = 0$ and $2(2\lambda - 1)x + 2(13 - \lambda)y + \lambda^2 - 9 = 0$

are perpendicular.

This gives $2 \times \left\{ -\dfrac{(2\lambda - 1)}{13 - \lambda} \right\} = -1$

or, $\lambda = 3$

Then the given equation becomes

$$(2x-y+3)^2 = 10x+20y = 10(x+2y)$$

or, $$\frac{(2x-y+3)^2}{2^2+1} = \frac{10}{\sqrt{5}} \frac{x+2y}{\sqrt{5}}$$

Taking the two perpendicular straight lines

$x+2y=0$ and $2x-y+3=0$ as the ~~axes~~

axes of coordinates, the formulae for transformation

are $$X = \frac{x+2y}{\sqrt{1+2^2}} \quad \text{and} \quad Y = \frac{2x-y+3}{\sqrt{2^2+1}}$$

and the given equation reduces to

$$Y^2 = \frac{10}{\sqrt{5}} X .$$

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2. Tangent and Normal

2.1 Equation of the tangent

Definition: If a line meets a curve at coincident points then the line is called the tangent to the curve at the meeting point and the point is called the point of contact.

Let $ax^2+2hxy+by^2+2gx+2fy+c=0$ ... (1)

be the equation of the conic and (x_1, y_1) be a point in the plane of the conic. The equation of a line through (x_1, y_1) can be written as

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = r \text{ (say)} \quad \cdots \quad (2).$$

where l and m are the cosines of the angles made by the line with x and y-axes respectively and r is the algebraical distance between (x, y) and (x_1, y_1) on the line.

From (2), $x = lr + x_1$, $y = mr + y_1$.

To find the point of intersection between (1) and (2), we have

$$a(lr+x_1)^2 + 2h(lr+x_1)(mr+y_1) + b(mr+y_1)^2 + 2g(lr+x_1) + 2f(mr+y_1) + c = 0$$

or, $$(al^2 + 2hlm + bm^2)r^2 + 2\{(ax_1+hy_1+g)l + (hx_1+by_1+f)m\}r$$
$$+ ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c = 0 \quad \cdots \quad (3)$$

It is a quadratic equation in r. Let the roots be r_1 and r_2 where r_1 and r_2 the distances of the two points of intersection between (1) and (2) from (x_1, y_1).

If the (x_1, y_1) is on the conic and the line (2) is the tangent at this point then $r_1 = 0$, and $r_2 = 0$

In this case $r_1 + r_2 = 0$ and $r_1 r_2 = 0$

By the equation (3)

$$(ax_1+hy_1+g)l + (hx_1+by_1+f)m = 0 \quad \cdots \quad (4)$$

and $ax_1^2 + 2hx_1y_1 + by_1^2 + 2gx_1 + 2fy_1 + c = 0 \quad \cdots (5)$

Eliminating l and m from (4) by (2)

$$(ax_1+hy_1+g)(x-x_1) + (hx_1+by_1+f)(y-y_1) = 0$$

or, $axx_1 + h(xy_1+yx_1) + byy_1 + gx + fy = ax_1^2 + 2hx_1y_1 + by_1^2 + gx_1 + fy_1$

$= -gx_1 - fy_1 - c \quad$ [by (5)]

or, $axx_1 + h(xy_1+yx_1) + byy_1 + g(x+x_1) + f(y+y_1) + c = 0 \quad \cdots (6)$

It is the standard form of the equation of the tangent at a point (x_1, y_1) on the conic.

Note: To write down the equation of the tangent at (x_1, y_1) to a conic the following rules are to be remembered:

Change x^2 into xx_1, y^2 into yy_1, $2xy$ into $xy_1 + yx_1$, $2x$ into $x+x_1$ and $2y$ into $y+y_1$.

Tangents of the standard equations

	Tangent at (x_1, y_1)
(1) Circle: $x^2+y^2+2gx+2fy+c=0$	$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$
$x^2+y^2=a^2$	$xx_1+yy_1=a^2$
(2) Parabola : $y^2=4ax$	$yy_1 = 2a(x+x_1)$
(3) Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$
(4) Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$
(5) Rectangular hyperbola: $xy=k^2$	$xy_1+yx_1=k^2$