$x = 1,$

$y = 2$

$z = -1$

So, the only solution of the system is

$x = 1, \; y = 2, \; z = -1$

Applications of linear systems: Nowadays in every field of science, we have to find a number of unknowns satisfying some conditions which can be transferred to equalities, and we get a system of linear equations there.

We now just describe an application in geometry:

Intersection two lines in Euclidean plane:

Let the lines be $a_{11}x + a_{12}y = b_1$

and $a_{21}x + a_{22}y = b_2$

Here the coefficient matrix is $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Augmented matrix is $[A, B] = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{bmatrix}$

Case 1 Rank of $A = 2$

There is a unique solution of the system since $\det A \neq 0$. So, the lines intersect at a point.

Case 2 Rank of $A = 1$ Rank of $[A, B] = 2$

The system is inconsistent and therefore there is no solution of the system. So, the lines are parallel

Case 3 Rank of A = 1, Rank [A, B] = 1

The system is consistent and there are infinite number of solutions since rank of $A < 2$

Then the two equations will be identical, i.e., the lines are coincident.

Examples 1. The line $2x+3y=3$ and $x+2y=1$ intersect at a point as the rank of $\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$ is 2

2. The lines $2x+3y=3$ and $4x+6y=7$ are parallel, since rank of the matrix $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ is 1 and rank of $\begin{pmatrix} 2 & 3 & 3 \\ 4 & 6 & 7 \end{pmatrix}$ is 2

3. The lines $2x+3y=3$ and $6x+9y=9$ are identical, since rank of the matrix $\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix}$

$= \text{rank of } \begin{pmatrix} 2 & 3 & 3 \\ 6 & 9 & 9 \end{pmatrix} = 1$

Intersection of two planes in three dimensional Euclidean space.

Let the planes be $a_{11}x + a_{12}y + a_{13}z = b_1$

and $a_{21}x + a_{22}y + a_{23}z = b_2$

Here the coefficient matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

and the augmented matrix $[A, B] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \end{bmatrix}$

Here for rank of $A = 2 =$ rank of $[A, B]$ we can show that planes ~~it~~ insect in a straight line.

and when rank $A = 1 =$ rank of $[A, B]$, the planes are coincident

and When rank of $A = 1$, rank of $[A, B] = 2$, the planes are parallel.

~~Simi~~ ~~Similarly~~ Similarly, taking more planes, we can discuss their intersections. ~~similar~~

Note: After reading vector space, the cases mentioned above can throughly discussed.

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§5. Linear difference equations with constant coefficients. (upto 2nd order).

5.1. Differences : The importance of role of finite differences and difference equations and the associated ~~matters~~ numerical methods has increased enormously to solve physical problems of practical interest. The advent of high ~~speed~~ speed computing machines has increase the importance of the changes in the dependent variable due to finite and discrete changes in the independent variable.

Let $y = f(x)$ be a function of a real variable x. If x is increased by a positive quantity h, then $f(x+h) - f(x)$ is called the difference of $f(x)$ with respect to x for the increment h, called the difference interval and the above difference is usually denoted by $\Delta f(x)$

Thus $\Delta f(x) = f(x+h) - f(x)$ ... (1)

This is also known as the finite difference of $f(x)$ to distinguish it from the small changes considered in the infinitesimal calculus.

The difference interval h is the finite jump in the independent variable

From (1), we see that Δ may be regarded as an operator which operating on function $f(x)$ gives the difference $f(x+h) - f(x)$.

Thus, for the difference interval h,

$$\Delta x^2 = (x+h)^2 - x^2 = 2xh + h^2$$

To denote the difference $f(x) - f(x-h)$, we use the operator ∇, such that $\nabla x^2 = x^2 - (x-h)^2 = 2xh - h^2$.

To distinguish between the two, we call Δ the forward difference operator and ∇ the backward difference