Unit 4 Coordinate Geometry

1.1 Transformation of Rectangular axes

Equation of curves or straight lines have reference to certain set of axes. In fact, we shall have different equations for the same curves when referred to different set of co-ordinate axes. This may happen when the origin is shifted to a point keeping the direction of axes the same or when the axes are rotated through the same angle keeping the origin unaltered. The former is called translation and the latter is called rotation. Change of co-ordinates may also be effected by a combination of the two, in either order, ~~is called~~ and is called a rigid motion. These transformations are also known as orthogonal transformations.

1.2 Change of origin without changing the direction of the axes (Translation)

Let OX, OY be the set of rectangular axes referred to which the co-ordinates of an arbitrary point P are (x,y), O is the

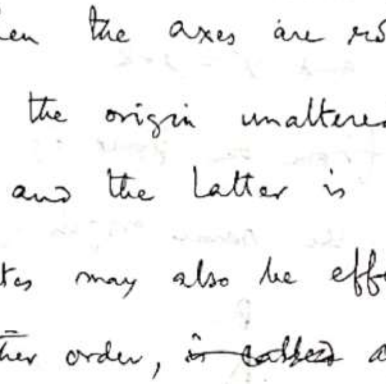


Fig - 1

origin. Let O', the new origin be at (h,k) and $O'X', O'Y'$ be the new set of axes parallel to the original axes, let the coordinates of P referred to the ~~new~~ new set of axes be (x',y'). Then $x = OM = OL + LM = OL + O'N = h + x'$

$$y = PM = MN + NP = O'L + PN = k + y'$$

Hence, $x = x' + h$, $y = y' + k$ are the transformation formulae for the translation of axes to the new origin (h, k)

Thus the equation $f(x, y) = 0$ with reference to the old set of axes becomes $f(x' + h, y' + k) = 0$ with reference to the new set of axes. Removing the primes to put it in general form, the new equation becomes $f(x + h, y + k) = 0$

Note: The above formulae can also be written as

$$x' = x - h \quad \text{and} \quad y' = y - k$$

1.3 Transformation from one pair of rectangular axes to another with the same origin (Rotation)

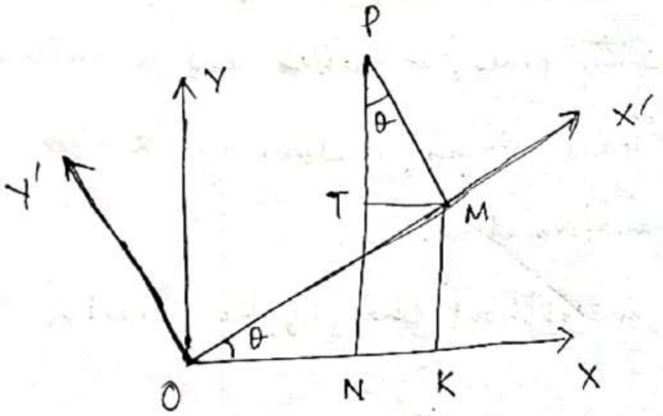


Fig. - 2

Let the axes OX and OY be turned about O through any angle θ to the position OX' and OY'. Let P be any point (x, y) referred to system OX, OY and (x', y') referred to the new set of axes OX', OY'.

PN and PM are drawn perpendiculars to OX and OX' respectively. Draw MK and MT perpendiculars to OX and PN respectively.

We have $OM = x'$, $PM = y'$. Then $x = ON = OK - NK$

$= OK - MT = x'\cos\theta - y'\sin\theta,$

since $\angle TPM = 90^\circ - \angle TMP = \angle TMO = \theta.$

Again $y = PN = TN + PT = MK + PT = x'\sin\theta + y'\cos\theta$

Hence
$$x = x'\cos\theta - y'\sin\theta \quad \ldots (1)$$
$$y = x'\sin\theta + y'\cos\theta \quad \ldots (2)$$

are the transformation formulae for rotation of axes.

note: Solving x' and y' from (1) & (2), we get

$$x' = x\cos\theta + y\sin\theta \quad \ldots (3)$$
$$y' = -x\sin\theta + y\cos\theta \quad \ldots (4)$$

The transformation of co-ordinates given (1), (2) and (3), (4) are expressed by the scheme

	x'	y'
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

1.4 <u>Translation followed by a rotation</u>

When the origin is shifted to the point (h, k), the co-ordinates of any point $P(x, y)$ become $(x+h, y+k)$. Then the axes are turned through an angle θ. The co-ordinates of P referred to the new set of axes are obtained by substituting $(x\cos\theta - y\sin\theta)$, for x and $(x\sin\theta + y\cos\theta)$ for y.

Hence the co-ordinates of P due to a rigid motion becomes $(h + x\cos\theta - y\sin\theta,\ k + x\sin\theta + y\cos\theta)$.

As is obvious, the result will be same if ~~rotation~~ rotation be followed by translation

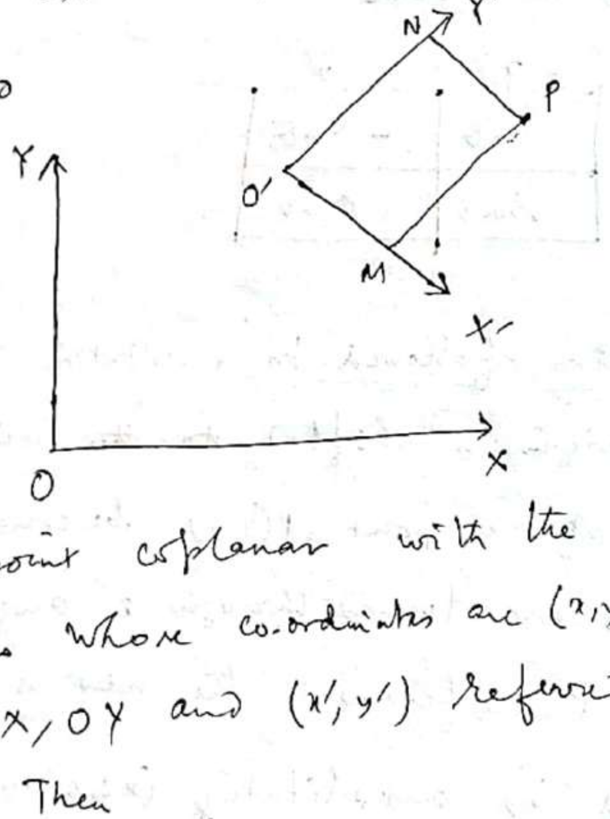
Corollary: The general formulae for transformation of axes may therefore be written as

$$x = px' - qy' + h, \quad y = qx' + py' + k \quad \text{where}$$

$$p^2 + q^2 = 1$$

1.5 General orthogonal transformations

Consider the system of rectangular axes, OX and OY, O being the origin. Let O' be the new origin of the new set of rectangular axes be O'X' and O'Y' where the equation of O'X' and O'Y' referred to the axes OX and OY be $ax+by+c=0$ and $bx - ay + c' = 0$



Let P be a point coplanar with the two system of axes whose co-ordinates are (x,y) referred to the set OX, OY and (x',y') referred to the set O'X', O'Y'. Then

$$x' = O'M = PN = \frac{bx - ay + c'}{\sqrt{a^2+b^2}} \quad \text{and} \quad y' = PM = \frac{ax+by+c}{\sqrt{a^2+b^2}}$$

Solving these two for x and y, we have

$$x = \frac{bx' + ay'}{\sqrt{a^2+b^2}} - \frac{ac + bc'}{a^2+b^2} \quad \text{and} \quad y = \frac{-ax' + by'}{\sqrt{a^2+b^2}} - \frac{bc - ac'}{a^2+b^2}$$