pair bisects the angles between the other pair, then prove that $pq = -1$.

Solution: The equation of the bisectors of the angles between the pair of straight lines $x^2 - 2pxy - y^2 = 0$ is

$$\frac{x^2 - y^2}{2} = \frac{xy}{-p}$$

that is, $px^2 + 2xy - py^2 = 0 \quad \ldots (1)$

But, by the condition of the problem, (1) is identical with

$$x^2 - 2qxy - y^2 = 0$$

comparing, we have $\frac{p}{1} = \frac{1}{-q} = \frac{p}{1}$

So, $pq = -1$

Q5. Show that the angle between one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ and one of the straight lines given by $ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$ is equal to the angle between the other two straight lines of the system.

Solution: The equation of the bisectors of the angles between the straight lines $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Also the equation of the bisectors of the angles between the straight lines $ax^2+2hxy+by^2+\lambda(x^2+y^2)=0$,

that is, $(a+\lambda)x^2+2hxy+(b+\lambda)y^2=0$

is
$$\frac{x^2-y^2}{(a+\lambda)-(b+\lambda)}=\frac{xy}{h}, \text{ that is } \frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

Thus the two pairs have the same bisectors and hence the problem follows.

6. Find the condition that one of the straight lines given by $ax^2+2hxy+by^2=0$ may coincide with one of the straight lines given by

$$a'x^2+2h'x'y'+b'y^2=0$$

Solution: Let the two pairs have a common straight line $y=mx$. Then substituting mx for y in the two equations and cancelling x^2, we have

$$a+2hm+bm^2=0$$
$$a'+2h'm+b'm^2=0$$

Therefore, by cross-multiplication,

$$\frac{m^2}{ah'-a'h}=\frac{2m}{ba'-b'a}=\frac{1}{hb'-h'b}$$

or, $4(ah'-a'h)(hb'-h'b)=(ba'-b'a)^2$

This is the required condition.

7. Show that the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$

and $lx+my+n=0$ is

$$\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$$

Solution: Let $ax^2+2hxy+by^2 = (l_1x+m_1y)(l_2x+m_2y)$

Equating the coefficients $l_1l_2 = a$, $l_1m_2+l_2m_1 = 2h$, $m_1m_2 = b$

The traingle is bounded by the lines

$$l_1x+m_1y = 0 \quad \ldots \quad (1)$$

$$l_2x+m_2y = 0 \quad \ldots \quad (2)$$

$$\text{and } lx+my+n=0 \quad \ldots \quad (3)$$

The lines (1) and (2) meet at (0,0). The lines (3) meets the lines (1) and (2) at

$$\left(\frac{-nm_1}{lm_1-ml_1}, \frac{nl_1}{lm_1-ml_1}\right) \text{ and } \left(\frac{-nm_2}{lm_2-ml_2}, \frac{nl_2}{lm_2-ml_2}\right)$$

The area of the triangle whose vertices are the above points

$$= \frac{1}{2}\left[\frac{-n^2l_2m_1+n^2l_1m_2}{(lm_1-ml_1)(lm_2-ml_2)}\right]$$

$$= \frac{1}{2}\frac{n^2(l_1m_2-l_2m_1)}{l^2m_1m_2-lm(l_1m_2+l_2m_1)+m^2l_1l_2}$$

$$= \frac{1}{2}\frac{n^2\sqrt{(l_1m_2+l_2m_1)^2-4l_1l_2m_1m_2}}{bl^2-2hlm+am^2}$$

$$= \frac{n^2\sqrt{h^2-ab}}{bl^2-2hlm+am^2}$$

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8. Show that the ~~tow~~ triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is right-angled, if $(a+b)(al^2+2hlm+bm^2)=0$

Solution: Let $ax^2+2hxy+by^2=(l_1x+m_1y)(l_2x+m_2y)$

Equating the coefficients $l_1l_2=a$, $l_1m_2+l_2m_1=2h$, $m_1m_2=b$

The lines are $l_1x+m_1y=0 \quad \ldots (1)$

$l_2x+m_2y=0 \quad \ldots (2)$

and $lx+my=1 \quad \ldots (3)$

The triangle will be right-angled, if any two of the above lines are at right angle.

The lines (1) and (2) will be at right angle if $l_1l_2+m_1m_2=0$

The lines (2) and (3) will be at right angle if $ll_2+mm_2=0$

The lines (3) and (1) will be at right angle if $ll_1+mm_1=0$

Combining these we have

$$(l_1l_2+m_1m_2)(ll_1+mm_1)(ll_2+mm_2)=0$$

or, $(a+b)\{l^2l_1l_2+lm(l_1m_2+l_2m_1)+m^2m_1m_2\}=0$

or, $(a+b)(al^2+2hlm+bm^2)=0$

9. Find the distance from the origin of the point of intersection of the pair of