chords through that point while the point itself is called the pole of the polar.

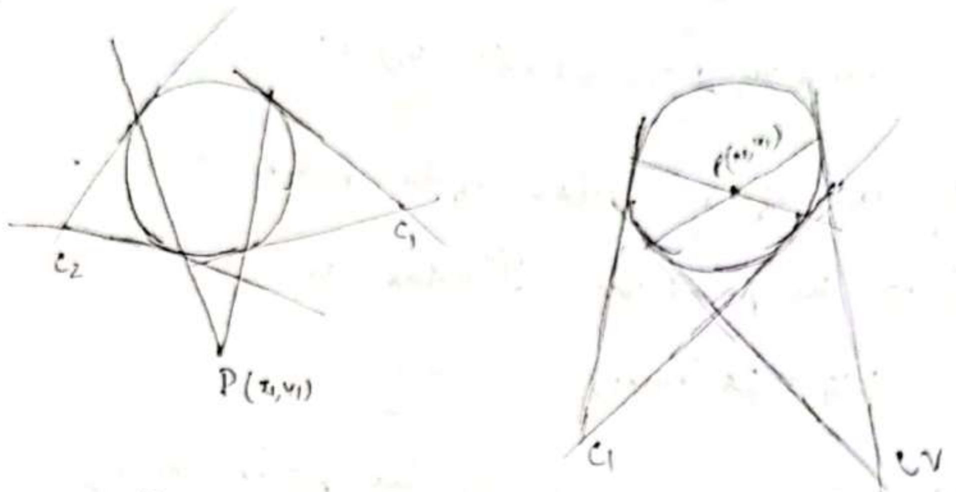

Figure 1 Figure 2

Let the equation of the conic be

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \quad \dots (1)$$

and (x_1,y_1) be the pole. Let the tangents at the extremities of a chord through (x_1,y_1) meet at (x_2,y_2). According to the definition of the polar, (x_2,y_2) lies on the polar of (x_1,y_1). The chord of contact of (x_2,y_2) is

$$axx_2+h(xy_2+yx_2)+byy_2+g(x+x_2)+f(y+y_2)+c=0$$

As (x_1,y_1) lies on it,

$$ax_1x_2+h(x_1y_2+y_1x_2)+by_1y_2+g(x_1+x_2)+f(y_1+y_2)+c=0$$

It shows that the locus of (x_2,y_2) is the line

$$axx_1+h(xy_1+yx_1)+byy_1+g(x+x_1)+f(y+y_1)+c=0 \quad \dots (2)$$

It is the polar of (x_1,y_1).

So, the equation of the polar of the point (x_1, y_1) with respect to the circle $x^2+y^2+2gx+2fy+c=0$

is $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$

The equation of the polar of the point (x_1, y_1) with respect to the parabola $y^2 = 4ax$ is

$$yy_1 = 2a(x+x_1)$$

The equation of the polar of the point (x_1, y_1) with respect to the ellipse ~~is~~ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

The equation of the polar of the point (x_1, y_1) with respect to the hyperbola ~~is~~ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

is $\cancel{\frac{xy}{a}}\ \cancel{\frac{xy}{a}}\ \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

Note The polar of a point with respect to a conic may be defined in another way.

Let A, B, C, D be four points on a line such that the ~~sg~~ segment AC, AB and AD are in H.P (Harmonic progression). In other words,

$\frac{1}{AC} + \frac{1}{AD} = \frac{2}{AB}$, that is, AB is the harmonic mean between AC and AD. The points C, D are said to be the harmonic conjugates of the point A, B and vice versa.

The locus of the harmonic conjugate of a fixed point P with respect to the two points in which any straight line through P cuts a given conic is called the polar of P with respect to the conic and P is called the pole of this polar.

The equation of the polar of the point (x_1, y_1) with respect to ~~[illegible]~~ any conic may also be obtained with the help of this definition.

For example, we find the equation of the polar of the point $P(x_1, y_1)$ with respect to the ~~conic~~ circle $x^2+y^2=a^2$ using this definition.

Any straight line through the point $P(x_1, y_1)$ is

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = r, \quad \text{where } l^2+m^2=1$$

For the points of intersection of the line and the circle, we have

$$(x_1+lr)^2+(y_1+mr)^2=a^2$$

$$\text{or, } r^2+2r(lx_1+my_1)+(x_1^2+y_1^2-a^2)=0$$

The roots r_1, r_2 of this quadratic equation in r give the directed distance of the points of intersection from the point (x_1, y_1). By the definition, the directed distance p of any point (α, β) on the polar is the harmonic mean between r_1 and r_2.

Hence $\frac{2}{p} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2} = \frac{-2(lx_1 + my_1)}{x_1^2 + y_1^2 - a^2}$ .. (1)

Also $\frac{\alpha - x_1}{l} = \frac{\beta - y_1}{m} = p$. So, $l = \frac{\alpha - x_1}{p}$ and

$m = \frac{\beta - y_1}{p}$. Therefore from (1), we get

$$2(x_1^2 + y_1^2 - a^2) = -2\left(x_1(\alpha - x_1) + y_1(\beta - y_1)\right)$$

or, $\alpha x_1 + \beta y_1 = a^2$

Hence the locus of (α, β) is $xx_1 + yy_1 = a^2$

This is the equation of the polar of P with respect to the circle.

Determination of the pole of a straight line with respect to a conic

(a) To find the pole of the straight line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = a^2$.

Let the pole of the given straight line be (x', y')

now the polar of (x', y') with respect to the circle $x^2 + y^2 = a^2$ is $xx' + yy' = a^2$

This is then identical with $lx + my + n = 0$