is $\beta y = 2a(x+\alpha) \quad \cdots \quad (2)$

comparing (1) and (2), we get

$$\frac{1}{\beta} = \frac{t}{-2a} = \frac{2at+at^3}{2a\alpha}$$

So, $t = \frac{-2a}{\beta}$, $2at+at^3 = \frac{2a\alpha}{\beta}$

Eliminating t, we get

$$2a\left(-\frac{2a}{\beta}\right) + a\left(-\frac{2a}{\beta}\right)^3 = \frac{2a\alpha}{\beta}$$

or, $\beta^2(\alpha+2a) + 4a^3 = 0$

Therefore the locus of (α,β) is $y^2(x+2a)+4a^3=0$.

5. Show that the locus of the poles of chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which subtend a right angle at the centre is $\frac{x^2}{a^4}+\frac{y^2}{b^4} = \frac{1}{a^2}+\frac{1}{b^2}$

Solution: Let $T(h,k)$ be the point of intersection of the tangents at P and Q. Thus the chord of contact PQ is $\frac{hx}{a^2}+\frac{ky}{b^2}=1 \quad \cdots \quad (1)$

of which $T(h,k)$ is the pole.

Now PQ subtends a right angle at the origin which is the centre of the ellipse. We now make the ~~given~~ equation of the given ellipse homogeneous with the help of (1) and

get $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \left(\frac{hx}{a^2} + \frac{ky}{b^2} \right)^2$

Since these two straight lines are at right angles,

therefore coefficient of x^2 + coefficient of y^2 = 0

or, $\left(\frac{h^2}{a^4} - \frac{1}{a^2} \right) + \left(\frac{k^2}{b^4} - \frac{1}{b^2} \right) = 0$

or, $\frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$

Hence ~~the equation~~ the locus of $T(h, k)$ is

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$$

6, Chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touch the circle $x^2 + y^2 = c^2$. Find the locus of their poles.

Solution: Let the pole of one chord be (h, k). Its polar with respect to the ellipse is

$\frac{xh}{a^2} + \frac{yk}{b^2} = 1$. This touches the circle

$x^2 + y^2 = c^2$. Therefore its perpendicular distance from the centre $(0, 0)$ is equal to the radius c

So, $\frac{1}{\sqrt{\frac{h^2}{a^4} + \frac{k^2}{b^4}}} = \pm c$. So, $\frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{c^2}$

Hence the locus of (h, k) is $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$

7. Show that the locus of the poles of tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$ is the parabola $y^2 = \frac{4b^2}{a}x$

Solution: Let (α, β) be the pole.

The polar of (α, β) with respect to $y^2 = 4bx$ is

$$y\beta = 2b(x+\alpha)$$

If it touches the parabola $y^2 = 4ax$, then we have

$$\frac{2b\alpha}{\beta} = \frac{a}{2b/\beta}, \text{ or, } \beta^2 = \frac{4b^2}{a}\alpha$$

Hence the locus of (α, β) is $y^2 = \frac{4b^2}{a}x$.

8. Show that the pole of any tangent to the hyperbola $xy = c^2$ with respect to the circle $x^2+y^2 = a^2$ lies on concentric and similar hyperbola.

Solution: The ~~equation~~ equation of the tangent to $xy = c^2$ at the point $(ct, c/t)$ is

$$x + t^2 y = 2ct \quad \dots (1)$$

Let (α, β) be its pole with respect to the circle.

The equation of the polar of (α, β) with respect to the circle is

$$x\alpha + y\beta = a^2 \quad \dots (2)$$

Equations (1) and (2) are identical

So, $\frac{\alpha}{1} = \frac{\beta}{t^2} = \frac{a^2}{2ct}$

Eliminating t, we have

$$\frac{a^2}{2c\alpha} = \frac{2c\beta}{a^2}$$

or, $$\alpha\beta = \frac{a^4}{4c^2}$$

Hence the locus of (α, β) is $xy = \frac{a^4}{4c^2}$, which is concentric and similar to $xy = c^2$

4.4. Polar equations of conics.

4.4.1 Polar coordinates

Let O and OX be a fixed point and a fixed line on a given plane. A point P on this plane can be defined with reference to the fixed point and the fixed line. If r be the distance of P from O and θ be the angular distance of P from X, i.e., $\angle POX = \theta$, then the polar coordinates of P are denoted by (r, θ). O and OX are called the pole and the initial line respectively. If (x, y) are the cartesian coordinates of P w.r.t OX and OY, then $x = r\cos\theta$ and $y = r\sin\theta$. r is known as the radius vector and θ is the vectorial angle of P.

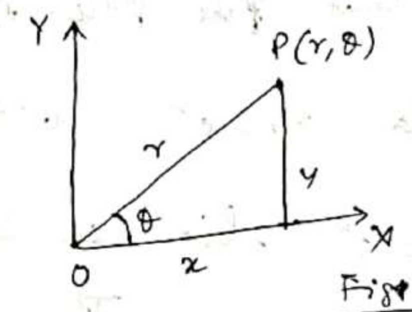

Fig 1