angle is geometrically measured in the anticlockwise direction.

1.5 <u>Distance between two points</u> (Fig. 2).

Let the polar coordinates of A and B be (r_1, θ_1) and (r_2, θ_2) respectively w.r.t the pole O and the initial line OX.

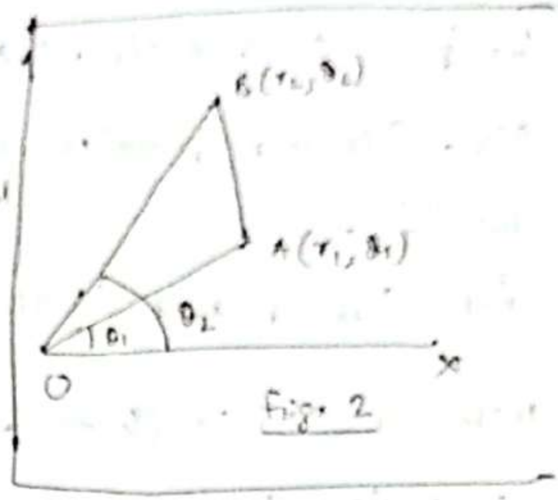


Fig. 2

In ΔOAB, $OA = r_1$, $OB = r_2$

and $\angle AOB = \theta_2 - \theta_1$

Now $AB^2 = OA^2 + OB^2 - 2OA.OB \cos\angle AOB$

$= r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)$

$\therefore AB = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}$

1.6 <u>Area of a triangle</u> (Fig. 3)

Let the polar coordinates of A, B and C be (r_1, θ_1), (r_2, θ_2) and (r_3, θ_3) respectively w.r.t the pole O and the initial line OX.

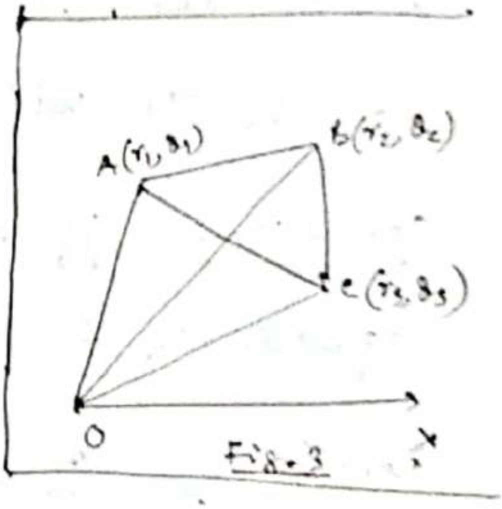


Fig. 3

From the fig. 3

$\Delta ABC = \Delta OAB + \Delta OBC - \Delta OCA$

$= \frac{1}{2} OA.OB \sin\angle AOB + \frac{1}{2} OB.OC \sin\angle BOC$

$- \frac{1}{2} OC.OA \sin\angle COA$

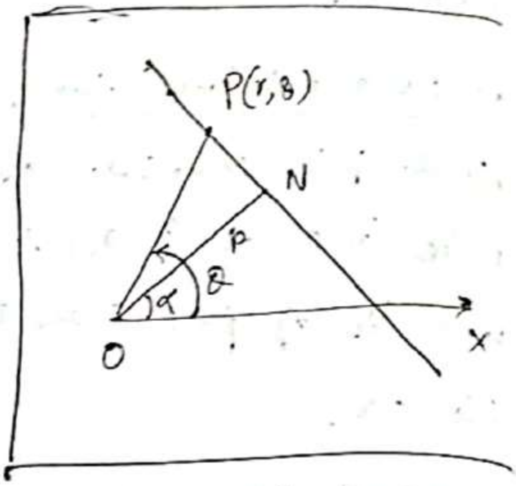
$= \frac{1}{2} (r_1r_2 \sin(\theta_1 - \theta_2) + r_2r_3 \sin(\theta_2 - \theta_3) - r_3r_1 \sin(\theta_1 - \theta_3)]$

$= \frac{1}{2} [r_1r_2 \sin(\theta_1 - \theta_2) + r_2r_3 \sin(\theta_2 - \theta_3) + r_3r_1 \sin(\theta_3 - \theta_1)]$

1.7 Polar equation of a straight line

Let (r, θ) be the coordinates of a point P on the line PN w.r.t the pole O and the initial line OX. ON is perpendicular to the line.

Let $ON = p$ and $\angle XON = \alpha$



Now $ON = OP \cos(\theta - \alpha)$ or, $p = r\cos(\theta - \alpha)$

It is the polar equation of the line.

Corollary 1 If $\alpha = 0$, $p = r\cos\theta$ is the equation of the line. It is a straight line perpendicular to OX, the initial line.

Corollary 2 If $\alpha = \pi/2$, $p = r\sin\theta$ is the equation of the line. It is parallel to OX.

Corollary 3 If $p = 0$, $\cos(\theta - \alpha) = 0$,

or $\theta - \alpha = \pi/2$ or $\theta = \text{constant}$. It is the equation of a line passing through the pole

Corollary 4 If the line passes through (r_1, θ_1) and makes an angle β with the initial line then $\beta = \pi/2 + \alpha$ and $p = r_1 \cos(\pi/2 - (\beta - \theta_1))$

or, $p = r_1 \sin(\beta - \theta_1)$.

The equation of the line is $r \sin(\beta - \theta) = r_1 \sin(\beta - \theta_1)$.

Corollary 5 The equation $r \cos(\theta - \alpha) = p$ can be written as $r \cos\theta \cos\alpha + r \sin\theta \sin\alpha = p$, or

$\frac{1}{r} = \frac{\cos\alpha}{p} \cos\theta + \frac{\sin\alpha}{p} \sin\theta$ which gives another form of the polar equation of the line as $\frac{1}{r} = A \cos\theta + B \sin\theta$, where A and B are constant. It is the general form of a straight line. The slope of the line is $-A/B$

Note: The polar equations of two parallel lines are of the form $r \cos(\theta - \alpha) = p$ and $r \cos(\theta - \alpha) = p'$.

The polar equations of two mutually perpendicular lines are of the form, $r \cos(\theta - \alpha) = p$ and $r \cos(\theta - \alpha - \pi/2) = p'$, i.e., $r \sin(\theta - \alpha) = p'$.

4.8. Polar equation of the line passing through (r_1, θ_1) and (r_2, θ_2)

The general form of the polar equation of a straight line is

$$A \cos\theta + B \sin\theta - \frac{1}{r} = 0 \quad \ldots (1)$$

If it passes through (r_1, θ_1) and (r_2, θ_2), then

$$A \cos\theta_1 + B \sin\theta_1 - \frac{1}{r_1} = 0 \quad \ldots (2)$$

and $$A \cos\theta_2 + B \sin\theta_2 - \frac{1}{r_2} = 0 \quad \ldots (3)$$

Eliminating A and from (1), (2) and (3), the required equation is

$$\begin{vmatrix} \cos\theta & \sin\theta & \frac{1}{r} \\ \cos\theta_1 & \sin\theta_1 & \frac{1}{r_1} \\ \cos\theta_2 & \sin\theta_2 & \frac{1}{r_2} \end{vmatrix} = 0$$

or, $\frac{1}{r}\sin(\theta_1-\theta_2) + \frac{1}{r_1}\sin(\theta_2-\theta) + \frac{1}{r_2}\sin(\theta-\theta_1) = 0$

Note: $A\cos\theta + B\sin\theta = \frac{k}{r}$ ... (1)

and $A\cos\theta + B\sin\theta = \frac{k_1}{r}$ ... (2)

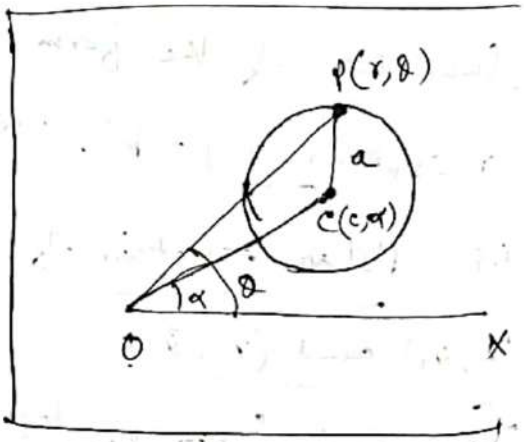
are parallel lines

$A\cos(\pi/2+\theta) + B\sin(\pi/2+\theta) = \frac{k_2}{r}$ i.e.,

$-A\sin\theta + B\cos\theta = \frac{k_2}{r}$ is perpendicular to (1) and (2)

4.9. Polar equation of a circle

Let (c, α) be the polar coordinates of the centre C of the circle of radius a w.r.t the pole O and the initial line OX. Let $P(r,\theta)$ be a point on the circle



From ΔOCP

$CP^2 = OC^2 + OP^2 - 2OC.OP\cos(\theta-\alpha)$

or, $a^2 = c^2 + r^2 - 2cr\cos(\theta-\alpha)$

or, $r^2 - 2cr\cos(\theta-\alpha) + c^2 - a^2 = 0$ ... (1)

It is the required equation.