matrix $[A, b] = \begin{bmatrix} 1 & a & a & 1 \\ a & 1 & 2a & -4 \\ a & -a & 4 & 2 \end{bmatrix}$

$|A| = (1+a)(2-a)^2$

So, $r(A) = 3$ when $a \neq -1$ and $a \neq 2$.

So, $r[A, b] = 3$ also when $a \neq -1$ and $a \neq 2$

So, when $a \neq -1$, and $a \neq -2$, $r(A) = r[A, b] = 3$

So, the system is consistent. As the number of variables is 3, so the system has unique solution when $a \neq -1$ and $a \neq 2$

By matrix method or Cramer's rule, we get

$$x = \frac{2(5a+2)}{(a-2)^2}, \quad y = \frac{8}{a-2}, \quad z = -\frac{7a-2}{(a-2)^2}$$

(Do the calculations yourself)

Next, when $a = -1$, $r(A) = 2 = r[A, b]$ (find it)

So, the system have an infinite number of solutions.

Lastly, when $a = 2$ $r(A) = 2$ and $r([A, b]) = 3$ (find it)

So, the system is inconsistent when $a = 2$.

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3. Investigate for what values of λ and μ, the following

equations
$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have (i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

Solution: The system is $AX = b$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $b = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$

The system has unique solution if $|A| \neq 0$

Here $|A| = \lambda - 3$

So, if $\lambda \neq 3$, the ~~s~~ system has a unique solution.

If $\lambda = 3$, the system has either no solution or infinitely ~~many~~ many solutions.

when ~~at~~ $\lambda = 3$, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$

and the augmented matrix $[A, b] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & 3 & \mu \end{bmatrix}$

Let us apply elementary row operations on $[A, b]$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & 3 & \mu \end{bmatrix} \xrightarrow{R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 0 & 0 & 0 & \mu-10 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & \mu-10 \end{bmatrix} \quad (1)$$

The equivalent system is $x + y + z = 6$
$y + 2z = 4$

If $\mu = 10$, then $r(A) = r[A, b] = 2 < 3 =$ number of variables, so, when $\lambda = 3$ and $\mu = 10$, the system have infinite number of solutions.

If $\mu \neq 10$, then $r(A) = 2 \neq r[A, b] = 3$

So, for $\lambda = 3$ and $\mu \neq 10$, the system has no solution.

Note: To get the solutions for $\lambda = 3$ and $\mu = 10$, put $3-2=1$ variable arbitrary value, say $z = c$, then $y = 4 - 2c$ and $x = 6 - 4 + 2c - c = 2 + c$, c is any real number. (from (1))

9. Solve the following system of homogeneous equations:

$$-7x+2y-3z=0$$
$$x-2y-3z=0$$
$$x+2y+5z=0$$

Solution: The coefficient matrix $A=\begin{bmatrix}-7 & 2 & -3\\ 1 & -2 & -3\\ 1 & 2 & 5\end{bmatrix}$

Let us apply elementary row operations on A

$$A \xrightarrow{R_3-R_2} \begin{bmatrix}-7 & 2 & -3\\ 1 & -2 & -3\\ 0 & 4 & 8\end{bmatrix} \xrightarrow{R_1+7R_2} \begin{bmatrix}0 & -12 & -24\\ 1 & -2 & -3\\ 0 & 4 & 8\end{bmatrix} \xrightarrow[\frac{1}{4}R_3]{-\frac{1}{12}R_1} \begin{bmatrix}0 & 1 & 2\\ 1 & -2 & -3\\ 0 & 1 & 2\end{bmatrix}$$

$$\xrightarrow{R_3-R_1} \begin{bmatrix}0 & 1 & 2\\ 1 & -2 & -3\\ 0 & 0 & 0\end{bmatrix} \xrightarrow{R_1\leftrightarrow R_2} \begin{bmatrix}1 & -2 & -3\\ 0 & 1 & 2\\ 0 & 0 & 0\end{bmatrix}$$

So, the system is equivalent to

$$x-2y-3z=0$$
$$y+2z=0$$

Here $r(A)=2<3=$ number of variables. So, we can put arbitrary values to $3-2=1$ variable.

Let $z=k$, k is a real number.

Then $y+2z=0$ gives $y=-2k$

and $x-2y-3z=0$ gives $x=2(-2k)+3k=-k$

So, the system have infinite number of solutions and given by $x=-k$, $y=-2k$ and $z=k$, k is a real number.

Exercises for the Tutorial (1)

[Solutions of these exercises are to be submitted within

January, 07, 2020 (07.01.2020). Make a pdf file by scanning the solutions pages and send it by whatsapp. If you need any help for these exercises, write me before 07.01.2020, I will send you some hints. Send the solutions positively ~~before~~ within the prescribed date.]

T1. Examine whether the the following systems are consistent or not:

(i) $x+y+z=9,\ 3x-2y+4z=3$

(ii) $2x+y+11=0,\ 6x+20y-6z+3=0,\ 6y-18z+1=0$

(iii) $2x-3y+7z=5,\ 5x-2y+4z=18,\ 2x+19y-47z=32$

T2. Find the values of k for which the system of linear equations
$$kx+y+z=1$$
$$x+ky+z=1$$
$$x+y+kz=1$$
will have (i) no solution (ii) a unique solution (iii) infinite number of solutions.

T3. Find the value of k such that the following system of linear equations is consistent:
$$2x+y-z=12$$
$$x-y-z=-3$$
$$3y+3z=k$$

T4. Find the value of k for which the system of equation
$$x+y+z=2$$
$$2x+y+3z=1$$
$$x+3y+2z=5$$
$$3x-2y+z=k$$
is solvable and then solve it.