Now we cover Unit 2 : Differential Calculus-I of the syllabus. The contents are :

• Rational numbers, Geometrical representations, Irrational number, Real number represented as point on a line – Linear Continuum. Acquaintance with basic properties of real number (No deduction or proof is included)

• Real-valued functions defined on an interval, limit of a function (Cauchy's Definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance (no proof) with the important properties of continuous functions on closed intervals. Statement of existence of inverse function of a strictly monotone function and its continuity.

• Derivative – its geometrical & physical interpretation. Sign of derivative – Monotonic increasing and decreasing functions. Relation between continuity and derivability. Differential – application in finding approximation.

• Successive derivative – Leibnitz's theorem and its application.

• Functions of two and three variables : Their geometrical representations. Limit and Continuity (definitions only) for functions of two variables. Partial derivatives. Knowledge and use of Chain Rule. Exact differentials (emphasis on solving problems only). Function of two variables – Successive partial derivatives : Statement of Schwarz's Theorem on Commutative property of mixed derivatives. Euler's Theorem on homogeneous function of two and three variables.

Rational numbers

The set $\{1, 2, 3, \ldots\}$ is called the set of natural numbers or set of all positive integers and is denoted by $\mathbb{N}$.

The set $\{0, 1, -1, 2, -2, \ldots\}$ is called the set of all integers and is denoted by $\mathbb{Z}$.

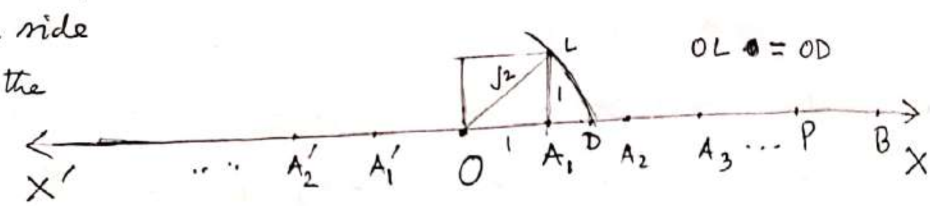
A rational number is of the form $\frac{p}{q}$ where p and q

are integers and $q \neq 0$. The set of all rational numbers is denoted by $\mathbb{Q}$. So, $\mathbb{Q} = \{ \frac{p}{q} : p, q \text{ are integers and } q \neq 0 \}$.

Here $\mathbb{N}$ is a proper subset of $\mathbb{Z}$ and $\mathbb{Z}$ is a proper subset of $\mathbb{Q}$.

Geometrical representation of rational numbers.

Rational numbers can be represented by points on a straight lines. Let $X'X$ be a straight line. We take a point ~~O~~ O on the line. O divides the line into two parts. The part of the right of O is called the positive side and the part ~~xx~~ to the left of O is called the ~~positive~~ negative side.



~~and~~ Let O represent the rational number 0 and take a point A_1 to the right of O such that A_1 represent the rational number one. Taking ~~o~~ the distance OA_1 as the unit distance on some chosen ~~sca~~ scale, each rational number can be represented by a unique point on the line. First of all, the positive integers $2, 3, \dots$ are represented by the points $A_2, A_3, \dots$ lying to the right of O where $OA_2 = 2OA_1$, $OA_3 = 3OA_1$ and the negative integers $-1, -2, \dots$ are represented by the points $A_1', A_2', \dots$ lying to the left of O such that $OA_1' = OA_1$, $OA_2' = 2OA_1, \dots$

To represent a rational number r of the form $\frac{p}{q}$ where p, q are positive integers, we measure p times the distance OA_1 to the right of O and get a point B and then measure the qth part of the distance OB to the right of O to get the point of O to get the point P. P represents the rational number r. If r be a negative rational number $(-s)$ then the point P' to the left of O (where $OP' = OP$ and P represents s). represents r. If all the rational numbers be plotted as points on the line it appears that the whole line is covered by rational points. A little further examination will show that such a view point is not true.

If we take a point D to the right of O such that OD is the length of the diagonal of the square on the side OA, then D is not a rational point (a point represented by a rational number) as $\sqrt{2}$ is not a rational number.

Let $r = \sqrt{2}$ or, $r^2 = 2$, we show that there is no rational r such that $r^2 = 2$. If possible, let r be a rational and let $r = \frac{p}{q}$ such that $\left(\frac{p}{q}\right)^2 = 2$ and assume p and q have no common factor other than 1. Now $\left(\frac{p}{q}\right)^2 = 2$

gives $p^2 = 2q^2 \Rightarrow p^2$ is even $\Rightarrow p$ is even

So, let $p = 2m$, m is an integer Then $p^2 = 2q^2 \Rightarrow 4m^2 = 2q^2$

$\Rightarrow q^2 = 2m^2 \Rightarrow q^2$ is even $\Rightarrow q$ is even. Since p and q are even, it has a common factor 2 other than 1 which contradicts our assumption that p and q have no common factor other than 1. So, there is no rational such that $r^2 = 2$. So, $\sqrt{2}$ is not a rational number.

So, the point D, therefore corresponds to a new type of number, called an irrational number. Every points other than rational points are represented by irrational numbers.

Real numbers : The set containing all rational as well as all irrational numbers is called the set of all real numbers and is denoted by $\mathbb{R}$.

Acquaintance with basic properties of real numbers

Algebraic properties of $\mathbb{R}$.

Addition and multiplication are defined on the set $\mathbb{R}$ satisfying the following properties:

A1. $a+b \in \mathbb{R}$ for all $a, b \in \mathbb{R}$

A2. $(a+b)+c = a+(b+c)$ for all $a, b, c \in \mathbb{R}$.

A3. There exists $0 \in \mathbb{R}$ such that $a+0 = 0+a = a$ for all $a \in \mathbb{R}$

A4. For each a in $\mathbb{R}$ There exist $-a \in \mathbb{R}$ such that

$a+(-a) = (-a)+a = 0$

A5. $a+b = b+a$ for all $a, b \in \mathbb{R}$

M1. $a.b \in \mathbb{R}$ for all $a, b \in \mathbb{R}$

M2. $(a.b).c = a.(b.c)$ for all $a, b, c \in \mathbb{R}$

M3. There exists $1 \in \mathbb{R}$ such that $a.1 = 1.a = a$ for all $a \in \mathbb{R}$.

M4. for each $a \neq 0$ in $\mathbb{R}$ there exists $\frac{1}{a}$ in $\mathbb{R}$ such that

$a.\frac{1}{a} = 1$

M5. $a.b = b.a$ for $a, b \in \mathbb{R}$.

D. $a.(b+c) = a.b + a.c$ for all $a, b, c \in \mathbb{R}$

Order properties on $\mathbb{R}$

On the set $\mathbb{R}$, a linear order relation $<$ is defined by "$a < b$ if $a, b \in \mathbb{R}$ and a is less than b" and it satisfies the following properties:

O1. If $a, b \in \mathbb{R}$, then exactly one of the following conditions holds: $a<b$ or $a=b$ or $b<a$ (law of trichotomy)

O2. $a<b$ and $b<c \Rightarrow a<c$ for $a, b, c \in \mathbb{R}$ (transitivity)

O3. $a<b \Rightarrow a+c < b+c$ for $a, b, c \in \mathbb{R}$

O4. $a<b$ and $0<c \Rightarrow ac < bc$ for $a, b, c \in \mathbb{R}$