Alternative method : we can do this conversion in the following way also:

$$\begin{array}{r|l}
2 & 79 \quad - 1 \\
2 & 39 \quad - 1 \\
2 & 19 \quad - 1 \\
2 & 9 \quad - 1 \\
2 & 4 \quad - 0 \\
2 & 2 \quad - 0 \\
 & 1
\end{array}$$

Hence $(79)_{10} = (1001111)$

2. Convert $(7181)_{10}$ from decimal to binary.

Solution:

$$\begin{array}{r|l}
2 & 7181 \quad - 1 \\
2 & 3590 \quad - 0 \\
2 & 1795 \quad - 1 \\
2 & 897 \quad - 1 \\
2 & 448 \quad - 0 \\
2 & 224 \quad - 0 \\
2 & 112 \quad - 0 \\
2 & 56 \quad - 0 \\
2 & 28 \quad - 0 \\
2 & 14 \quad - 0 \\
2 & 7 \quad - 1 \\
2 & 3 \quad - 1 \\
 & 1
\end{array}$$

So, $(7181)_{10} = (1110000001101)_2$

3. Convert $(10010011)_2$ from binary to decimal.

Solution: $(10010011)_2 = 1.2^7 + 0.2^6 + 0.2^5 + 1.2^4 + 0.2^3 + 0.2^2 + 1.2 + 1$

$= 128 + 16 + 2 + 1 = (147)_{10}$

4. Convert $(9205)_{10}$ from decimal to base 6 notation

Solution:

$$\begin{array}{r|l}
6 & 9205 \quad - 1 \\
6 & 1534 \quad - 4 \\
6 & 255 \quad - 3 \\
6 & 42 \quad - 0 \\
6 & 7 \quad - 1 \\
 & 1
\end{array}$$

Hence $(9205)_{10} = (110341)_6$

5. Convert $(2FB5)_{16}$ from hexadecimal to binary

Solution:

$(2FB5)_{16} = 2.16^3 + F.16^2 + B.16 + 5$

$= 2.16^3 + 15.16^2 + 11.16 + 5$

(since in hexadecimal ~~notion~~ notation F represents 15 and B represents 11)

$= 2.2^{12} + 15.2^8 + 11.2^4 + 5$

$= (2.1+0)2^{12} + (2^3+2^2+2+1).2^8 + (2^3+2+1).2^4 + (2^2+1)$

(expressing 2, 15, 11 and 5 in binary)

$= 2^{13} + 0.2^{12} + 1.2^{11} + 1.2^{10} + 1.2^9 + 1.2^8 + 1.2^7 + 0.2^6 + 1.2^5 + 1.2^4$
$+ 0.2^3 + 1.2^2 + 0.2 + 1$

$= (10111110110101)_2$

6. Convert $(10111110110011)_2$ from binary to hexadecimal.

Solution:

$(10111110110011)_2 = 1.2^{13} + 0.2^{12} + 1.2^{11} + 1.2^{10} + 1.2^9 + 1.2^8 + 1.2^7 + 0.2^6 + 1.2^5 + 1.2^4$
$+ 0.2^3 + 0.2^2 + 1.2 + 1$

$= 2^{13} + 2^{11} + 2^{10} + 2^9 + 2^8 + 2^7 + 2^5 + 2^4 + 2 + 1$

$= (2^4)^3.2 + (2^4)^2.2^3 + (2^4)^2.2^2 + (2^4)^2.2 + (2^4)^2 + 2^4.2^3 + 2^4.2 + 2^4 + 2 + 1$

$= 2.16^3 + 8.16^2 + 4.16^2 + 2.16^2 + 16^2 + 8.16 + 2.16 + 16 + 2 + 1$

$= 2.16^3 + 15.16^2 + 11.16 + 3$

$= (2FB3)_{16}$ (since in hexadecimal notation F represents 15 and B represents 11)

Exercise: 1. Convert $(3199)_{10}$ from decimal to base 2 notation.

2. Convert $(5554)_7$ from base 7 to decimal representation.

3. Convert $(10010001)_2$ from binary to decimal notation.

4. Convert $(CDEF)_{16}$ from hexadecimal to binary notation.

5. Convert $(1011110011001 1)_2$ from binary to hexadecimal notation.

6. Convert $(3DE9)_{16}$ from hexadecimal to decimal representation.
7. Convert $(31572)_{10}$ from ~~hexadecimal~~ decimal to hexadecimal.

Prime integers: An integer $p > 1$ is called a prime number or simply a prime if the only positive divisor of p are 1 and p. A integer $q > 1$ which is not a prime is called composite.

Examples: 2, 3, 5, 7 are all prime numbers and 4, 6, 8, 9 are composite numbers.

Theorem 9 Any integer $p > 1$ is a prime number if and only if p divides ab implies either p divides a or p divides b where a, b are any two integers.

Proof: Suppose p is a prime and a, b are two integers such that $p \mid ab$. If $p \mid a$, then we are done. Suppose p does not divide a. Since 1 and p are the only positive divisors of p, it follows that $\gcd(p, a) = 1$. Hence there exist integers r and t such that $1 = rp + ta$. Then $b = brp + tab$. Now $p \mid brp$ and $p \mid t(ab)$. Hence $p \mid b$

Conversely, Suppose that the integer p satisfies the condition. Let q be a positive divisor of p such that $q < p$. We show that $q = 1$. Now there exists an integer r such that $p = qr$. Since p divides p, we find that p divides qr. Hence either p divides q or p divides r. Since $0 < q < p$, p does not divide q. So, p divides r. Then $r = pt$ for some integer t. Hence $p = qr = qpt$. This implies $qt = 1$. Hence $q = 1$. So, we conclude that 1 and p are the only divisors of p. Hence p is a prime.

<u>Corollary 1</u> For any integer $n \geq 2$, if p divides $a_1 a_2 \cdots a_n$, then p divides one of the integers $a_1, a_2, \ldots, a_n$.

Proof: We prove this result by induction on n. For $n=2$, the result follows from Theorem 9. Suppose the result holds for some integer $k \geq 2$. Consider now the product $a_1 a_2 \cdots a_k a_{k+1}$ of $k+1$ integers and assume $p \mid a_1 a_2 \cdots a_k a_{k+1}$. Then $p \mid (a_1 a_2 \cdots a_k) a_{k+1}$. Hence from Theorem 9 either $p \mid a_1 a_2 \cdots a_k$ or $p \mid a_{k+1}$. If p divides $a_1 a_2 \cdots a_k$, then by induction hypothesis, p divides one of $a_1, a_2, \ldots, a_k$. So, we conclude that p divides one of $a_1, a_2, \ldots a_k, a_{k+1}$. Hence by induction the result follows.

<u>Example</u> Consider the integer 12. 12 divides 8×3. But neither 8 nor 3 is divisible by 12. Hence 12 is not a prime.

<u>Theorem 10</u>: Every integer $n \geq 2$ has a prime factor.

Proof: We prove this result by strong form of mathematical induction.

For $n=2$, 2 is a prime factor of n. Hence the result holds for $n=2$. Assume that each of the integers $2, 3, \ldots, k-1$ has a prime factor. Now, consider the integer $k > 2$. If k itself is a prime then k has a prime factor. If k is not prime, then k is composite and hence there exist integers r and s such $k = rs$, where $1 < r < k$ and $1 < s < k$

Since $2 \leq r < n$, by induction hypothesis r has a prime factor which is also a prime factor of k. Thus it follows that k has a prime factor. So, by mathematical induction, every integer $n \geq 2$ has a prime factor.

Theorem 11 (Euclid's theorem) There are infinite number of primes.

Proof: The integers 2, 3, 5, 7, 11, 13 are primes. Suppose there are finite number of primes and let there be only n distinct primes. Let $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, $p_5 = 11$, $p_6 = 13, \ldots, p_n$ be the n primes.

Let $m = p_1 p_2 \cdots p_n + 1$. Now $m > 1$. ~~Since m is not divisible by p_i, $i = 1, 2, \ldots, n$~~ Hence by Theorem 10 we find that m has a prime factor p (say). Since m is not divisible by p_i, $i = 1, 2, \ldots, n$, it follows that p is different from $p_1, p_2, \ldots, p_n$. This contradicts our assumption that $p_1, p_2, \ldots, p_n$ are the only primes. So, we conclude There exist infinitely many primes.

Theorem 12 Let n be a positive integer greater than 1. If n is not a prime, Then n has a prime factor not exceeding $\sqrt{n}$

Proof: Since $n > 1$ and n is not a prime, there exist integers r and t such that $1 < r \leq t < n$ and $n = rt$. Then we must have, $r \leq \sqrt{n}$. For, if $r > \sqrt{n}$, then $t \geq r > \sqrt{n}$ implies $n = rt > \sqrt{n} \cdot \sqrt{n} = n$ and this leads to a contradiction. From Theorem 10, we find that r has a prime factor p (say), which is also a prime factor of n. Clearly $p \leq \sqrt{n}$.