Solution: ~~Let us construct the truth table for~~
$f(x,y,z) = (x+y+z)(xy+x'z)'$

Row	x	y	z	x'	$x+y+z$	xy	$x'z$	$xy+x'z$	$(xy+x'z)'$	$f(x,y,z)$
1	1	1	1	0	1	1	0	1	0	0
2	1	1	0	0	1	1	0	1	0	0
3	1	0	1	0	1	0	0	0	1	1
4	1	0	0	0	1	0	0	0	1	1
5	0	1	1	1	1	0	1	1	0	0
6	0	1	0	1	1	0	0	0	1	1
7	0	0	1	1	1	0	1	1	0	0
8	0	0	0	1	0	0	0	0	1	0

Next we consider only those rows in which the value of $f(x,y,z)$ is 1. Here those rows are 3, 4 and 6. For each of these rows, we construct minterms.

For the 3rd row, the minterm $= xy'z$,
For the 4th row, the minterm $= xy'z'$,
For the 6th row, the minterm $= x'yz'$,

Hence $(x+y+z)(xy+x'z)' = xy'z + xy'z' + x'yz'$ which is in DNF in x, y, z

Alternative method:

$$(x+y+z)(xy+x'z)'$$
$$= (x+y+z)\left((xy)'(x'z)'\right)$$
$$= (x+y+z)\left((x'+y')(x+z')\right)$$
$$= (x+y+z)(x'x + x'z' + y'x + y'z')$$
$$= (x+y+z)(0 + x'z' + y'x + y'z')$$
$$= (x+y+z)(x'z' + y'x + y'z')$$
$$= xx'z' + xy'x + xy'z' + yx'z' + yy'x + yy'z' + zx'z' + zy'x + zy'z'$$
$$= 0 + xy' + xy'z' + x'yz' + 0 + 0 + 0 + xy'z + 0$$

(Since $xx' = 0$ etc. and $xx = x$ etc.)

$= xy' + xy'z' + x'yz' + xy'z$

$= xy'(z+z') + xy'z' + x'yz' + xy'z$

[Since $z+z'=1$ and $x.1=x$.]

$= xy'z + xy'z' + xy'z' + x'yz' + xy'z$

$= xy'z + xy'z' + x'yz'$. [$\because x+x = x$ etc.]

which is the required DNF.

3. Consider the Boolean function $f(x,y,z)$ given by the following truth table

Rows	x	y	z	$f(x,y,z)$
1	1	1	1	1
2	1	0	0	0
3	1	1	1	0
4	1	0	0	1
5	0	1	1	1
6	0	0	0	0
7	0	1	1	0
8	0	0	0	0

Find the Boolean function $f(x,y,z)$ in DNF

Solution: The function $f(x,y,z)$ takes the value 1 for the assignments in the 1st, 4th and 5th rows of the given table. The min terms corresponding to 1st, 4th and 5th terms are xyz, $xy'z'$ and $x'yz$.

So, $f(x,y,z) = xyz + xy'z' + x'yz$ is in DNF.

CONJUNCTIVE NORMAL FORM

We now consider another form a Boolean function or expression, called conjunctive normal form (CNF) which is equally important and useful: In a

DNF the expression is a sum of products whereas in a CNF, the expression is a product of sums.

Definition: A Boolean function or expression α in the variables $x_1, x_2, \ldots, x_n$ is called a maxterm if

$$\alpha = \tilde{x}_1 + \tilde{x}_2 + \cdots + \tilde{x}_n \text{ where each } \tilde{x}_i \text{ denotes } x_i \text{ or } x_i'.$$

Definition: A Boolean function or expression $\alpha(x_1, x_2, \ldots, x_n)$ is said to be in conjunctive normal form (CNF) if there exist distinct maxterms $\alpha_1, \alpha_2, \ldots, \alpha_m$ in the variables $x_1, x_2, \ldots, x_n$ such that

$$\alpha = \alpha_1 \alpha_2 \cdots \alpha_m$$

Examples: The Boolean expressions

$$(x+x'),\ (x+y)(x+y')(x'+y),\ (x'+y+z)(x'+y+z')(x'+y'+z')$$

in one, two and three variables respectively are in CNF, whereas the Boolean expressions $(xy'+xz)'+x'$, $x+x'y$ are neither in CNF ~~or~~ nor in DNF.

To express ~~to~~ a Boolean expression in CNF, we find its truth table, and consider only those rows for which the value of α is 0. ~~For~~ From each of these rows we find the maxterm in the following way: The max term for a row is

$$\tilde{x}_1 + \tilde{x}_2 + \dots + \tilde{x}_n$$

satisfying the condition $\tilde{x}_i = \begin{cases} x_i & \text{if } x_i = 0 \\ x_i' & \text{if } x_i = 1 \end{cases}$

Then take the products of all those max terms to get the CNF

Example 1 Express the boolean expression $(x+y+z)(xy+x'z)'$ in CNF.

Solution: we first find out the truth table of

$\alpha(x,y,z) = \cancel{(x+y+z)(xy+x'z)}' \; (x+y+z)(xy+x'z)'$

Rows	x	y	z	x'	$x+y+z$	xy	$x'z$	$xy+x'z$	$(xy+x'z)'$	$\alpha(x,y,z)$
1	1	1	1	0	1	1	0	1	0	0
2	1	1	0	0	1	1	0	1	0	0
3	1	0	1	0	1	0	0	0	1	1
4	1	0	0	0	1	0	0	0	1	1
5	0	1	1	1	1	0	1	1	0	0
6	0	1	0	1	1	0	0	0	1	1
7	0	0	1	1	1	0	1	1	0	0
8	0	0	0	1	0	0	0	0	1	0

Here the rows in which the value of $\alpha(x,y,z)$ is 0 are 1, 2, 5, 7 and 8.

For row 1, the max term $= x'+y'+z'$

For row 2, the max term $= x'+y'+z$

For row 5, the max term $= x+y'+z'$

For row 7, the max term $= x+y+z'$

For row 8, the max term $= x+y+z$

So, $\alpha(x,y,z) = (x'+y'+z')(x'+y'+z)(x+y'+z')(x+y+z')(x+y+z)$ is in CNF in x, y, z