Alternative method:

$(x+y+z)(xy+x'z)' = (x+y+z)\left((xy)'(x'z)'\right)$

$= (x+y+z)\left((x'+y')(x+z')\right)$

$= (x+y+z)\left(\left((x'+y')+(zz')\right)\left((x+z')+(yy')\right)\right)$

$= (x+y+z)\left((x'+y'+z)(x'+y'+z')\right)\left((x+z'+y)(x+z'+y')\right)$

$= (x+y+z)(x'+y'+z)(x'+y'+z')(x+y+z')(x+y'+z')$

The last expression is in CNF.

Example 2 Express the following Boolean expression in CNF in the variables present in the expression: $x'+yz$

Solution $x'+yz = (x'+y)(x'+z)$

$= (x'+y+zz')(x'+z+yy')$

$= (x'+y+z)(x'+y+z')(x'+z+y)(x'+z+y')$

$= (x'+y+z)(x'+y+z')(x'+y+z)(x'+y'+z)$

is in CNF in the variables x, y, z

Application of Boolean Algebra: Switching Circuits:

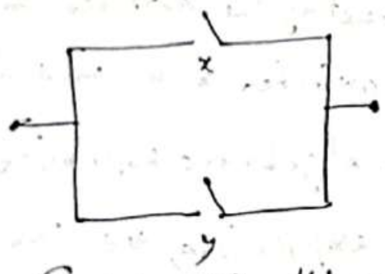
A switch is a mechanical device attached to a point on a wire having only two possible states - on or off, i.e., closed or open. The switch allows current to flow through the point when it is on in the on state and no current can flow through the point when it is in off state. We shall denote a switch by a variable like $x, y, \ldots$, where the variable can assume only two values 1 and 0. The variable x representing the switch takes the value 1 when the switch is on (or, closed) and the value 0 when it is off (or, open). The switch x is geometrically represented as . If two points are connected by wires on which there are finite number of switches, we say that the points are connected by a switching circuit.

Definition: Equivalent switching circuits: Two switching circuits are said to be equivalent if current flows through one when and only when it flows through the other. In other words, two circuits are said to be equivalent if and only if they have the same closure conditions for any given state of the switches involved in the circuits.

Definition: Simpler Circuit: One circuit is said to simpler than another circuit if the first circuit contains fewer switches than the other circuit. If two or switches operate in such a way that they are in the same state (i.e., on or off) simultaneously, we shall denote them by the same variable. Again if they operate in such a way that when one switch is on, the other is off and vice versa, we shall denote one of them by a variable, say x, and the other by x'.

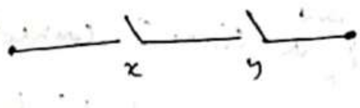
We denote a switching circuit having two switches connected by wires in parallel by $x+y$ and if they ~~are~~ ~~con~~ are connected by wires in series we denote the circuit by $x.y$ or simply xy.

The geometrical representations are :



Switches in parallel : $x+y$

Figure 1

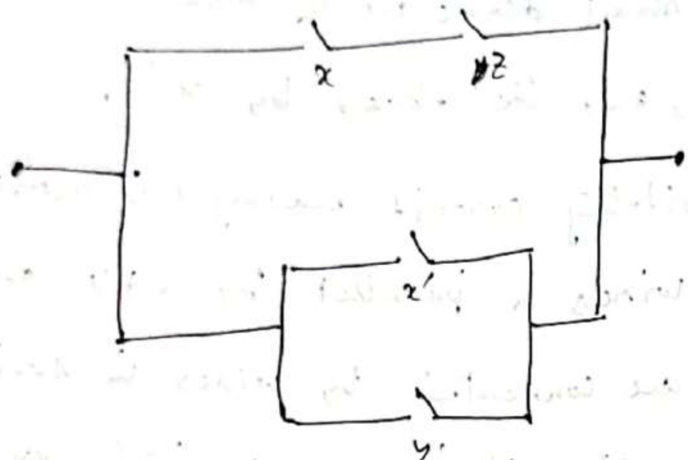


(Switches in series : xy)

Figure 2

We now consider the switching circuit given in figure 3. In this circuit, the switches x and z are in series and switches x' and y are in parallel and then the first pair of switches are in ~~series~~ parallel with the second pair of

Switches. Figure 3 shows that we may combine switches in parallel and in series in the same circuit. Such a circuit is called a series-parallel ~~circuit~~ switching circuit. In our discussion we shall confine ourselves only in series-parallel circuits including series and parallel circuits



(Series-parallel switching circuit)

Figure 3

We have seen that a circuit connecting the switches in parallel and a circuit connecting the switches x, y in series are represented respectively by the algebraic expressions $x+y$ and xy. Using ~~these~~ these expressions, we can find an algebraic expression, similar to a Boolean ~~expression involving variables and +, ·, ' only~~, corresponding expression, for each series-parallel circuit. Conversely to each algebraic expression involving ~~vars~~ variables and +, ·, ' only, there corresponds a switching circuit. We then say that the expression represents the circuit and the circuit realizes the expression.