For example, the circuit given in Figure 3 can be represented by the algebraic expression $xz + (x' + y)$. Again, the circuit given in Figure 4 can be represented by the algebraic expression $xy + (z+x)y'$. Two algebraic expressions corresponding to two circuits are said to be equivalent if and only if they represents equivalent circuits.

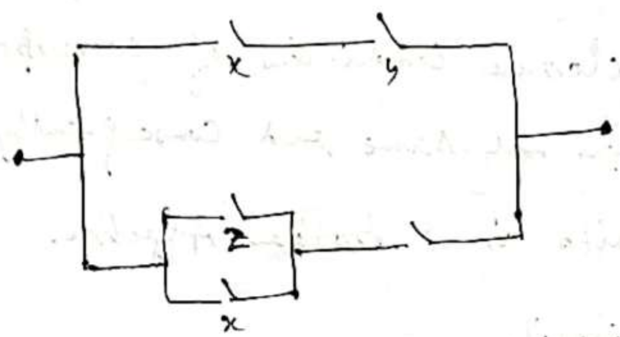

Figure 4

The variables present in the expression representing a circuit can take only two values, 1 and 0 since the switches can ~~only~~ have only two states — on and off. Hence the expression determines a function, called switching function, which gives the closure condition of the circuit, i.e., whether the circuit is closed or open under a given set of states of the switches.

In accordance with the laws for the flow of current through circuits containing two switches in parallel and in series, the closure conditions of the switching functions

$x+y$, xy and x' are as follows

x	y	$x+y$	xy	x'
1	1	1	1	0
1	0	1	0	0
0	1	1	0	1
0	0	0	0	1

We can easily verify all the fundamental laws of Boolean algebra are valid in the algebra of circuits by showing that closure conditions of corresponding switching functions are same and consequently, the algebra of circuits is a Boolean Algebra.

Simplification of Circuits

We have seen that the algebra of circuits is a Boolean Algebra. Hence the theorems and laws used in simplification of Boolean expressions apply in the algebra of circuits. To find an equivalent simplified circuit for a given ~~ckt~~ circuit, we first find the Boolean expression which represents the circuit. Then we simplify the expression using the laws of Boolean Algebra and finally draw a new simpler circuit which realizes the simplified expression.
We ~~is~~ illustrate the method by the following example:

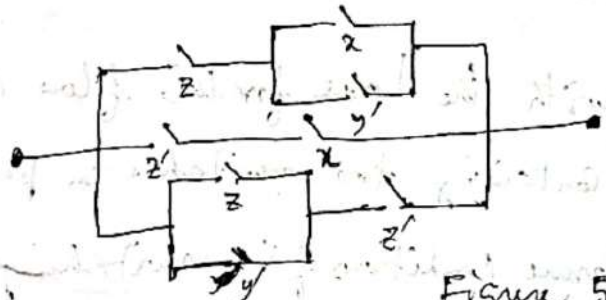

Figure 5

To find a simpler equivalent circuit for that given in the figure 5, we proceed as follows:

The given circuit is represented by the Boolean expression $z(x+y')+z'x+(z+y')z'$. This is equivalent to

$$zx+zy'+z'x+zz'+y'z' = (zx+z'x)+(zy'+y'z')+0$$
$$= x(z+z')+y'(z+z')$$
$$= x\cdot 1 + y'\cdot 1$$
$$= x+y'$$

Hence the given circuit in figure 5 is equivalent to a simpler circuit shown in figure 6

Figure 6

Worked out examples

1. Find a switching circuit which realizes the Boolean expression $x(y(z+w)+z(u+v))$.

Solution: The expression indicates a series connection of x and $y(z+w)+z(u+v)$. Again $y(z+w)+z(u+v)$ consists of parallel connection of $y(z+w)$ and $z(u+v)$. Then $y(z+w)$ consists y in series with a parallel connection of z and w. Similarly $z(u+v)$ consists of z in series with a parallel connection of u and v. Hence the Boolean expression is represented by the following circuit in Figure 7

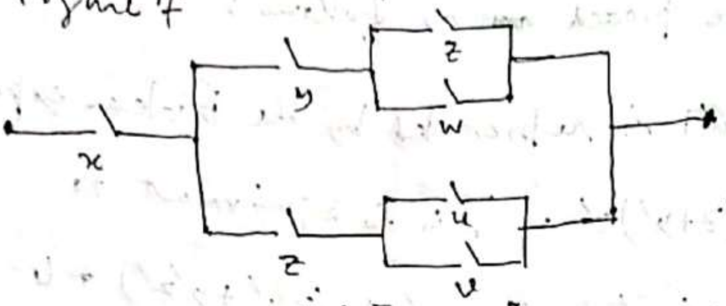

Figure 7

2. Find the Boolean expression which represents the following circuit in Figure 8.

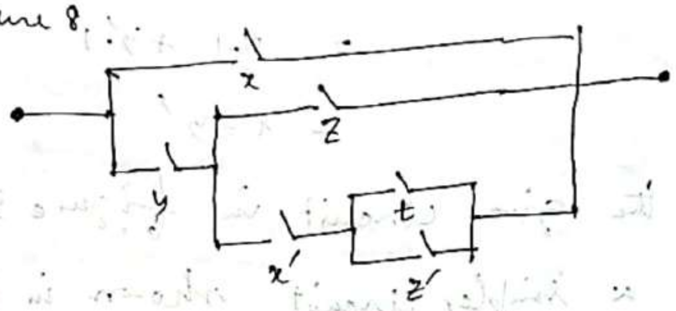

Figure 8

Solution: From figure 8, by inspection, we get the Boolean expression ~~rep~~ ~~represents~~ representing the circuit as

$$x + y\left(z + x'(t+z')\right).$$

3. Construct the switching table for the switching function f represented by the Boolean expression $xyz + x'(y+z)$:

Solution: Let $f(x,y,z) = xyz + x'(y+z)$

x	y	z	x'	xy	xyz	$y+z$	$x'(y+z)$	$f(x,y,z)$
1	1	1	0	1	1	1	0	1
1	1	0	0	1	0	1	0	0
1	0	1	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0
0	1	1	1	0	0	1	1	1
0	1	0	1	0	0	1	1	1
0	0	1	1	0	0	1	1	1
0	0	0	1	0	0	0	0	0

The table for the switching function f consists of the columns under the