Now mean of the given frequency distribution is

$$x = \frac{\sum f_i x_i}{n}, \text{ where } n = \sum f_i = 40$$

$$= \frac{439}{40} = 10.975$$

Example 3.5

Find the mean of the following frequency distribution:

Age (in years) :	40-44	45-49	50-54	55-59	60-64
No. of persons :	17	25	30	20	8

Table 4.2

Necessary Calculation for mean

Age (in years)	No. of persons (f_i)	Class marks (x_i)	$y_i = \frac{x_i - 52}{5}$	$f_i y_i$
40 – 44	17	42	−2	−34
45 – 49	25	47	−1	−25
50 – 54	30	52	0	0
55 – 59	20	57	1	20
60 – 64	8	62	2	16
	100	—	—	−23

Here $\bar{y} = \frac{\sum_i f_i y_i}{n}$ where $n = \sum_i f_i$

$$= \frac{-23}{100} = -0.23$$

Since $y_i = \frac{x_i - 52}{5}$ or, $x_i = 52 + 5y_i$

So, $\bar{x} = 52 + 5\bar{y} = 52 + 5(-0.23) = 50.85$

The arithmetic mean has several properties and some of them are as follows:

(a) If the observed values of a variable are all equal, then their mean will be the common value.

Suppose we are given n values $x_1, x_2, \ldots, x_n$ of a variable x, where $x_i = c$ for each i.

Then $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} c = nc$ Hence $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{nc}{n} = c$

(b) The sum of the deviations of the values of a variable from its mean is zero.

<u>First case</u>: Suppose a variable x assumes n values $x_1, x_2, \ldots, x_n$ has mean $\bar{x}$, where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$.

clearly, $\sum_{i=1}^{n} x_i = n\bar{x}$ now $\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - n\bar{x}$

$= n\bar{x} - n\bar{x} = 0$

<u>Second Case</u>: In connection with frequency distribution of a discrete variable x, Suppose $f_1, f_2, \ldots, f_r$ denotes the frequencies corresponding to the values $x_1, x_2, \ldots, x_r$ respectively.

Then $\bar{x} = \frac{1}{n}\sum_{i=1}^{r} f_i x_i$ where $n = \sum_{i=1}^{r} f_i$

or, $\sum_{i=1}^{r} f_i x_i = n\bar{x}$.

now $\sum_{i=1}^{r} f_i(x_i - \bar{x}) = \sum_{i=1}^{r} f_i x_i - \bar{x}\sum_{i=1}^{r} f_i$

$= n\bar{x} - n\bar{x} = 0$

(e) Suppose x is a linear function of y in the form $x = a + by$, then the arithmetic mean of x and y are related as $\bar{x} = a + b\bar{y}$

Here $x = a + by$

or, $x_i = a + by_i$ for each i

or $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (a + by_i)$, where n denotes the number of given values

or $\sum_{i=1}^{n} x_i = na + \sum_{i=1}^{n} y_i$

or, $\frac{1}{n} \sum_{i=1}^{n} x_i = a + \frac{1}{n} \sum_{i=1}^{n} y_i$

or, $\bar{x} = a + b\bar{y}$

This result shows that the arithmetic mean is affected by change of both origin (here a) and scale (here b)

Example 3.6

The arithmetic mean of a variable y is 35. Find the mean of the variable $3y - 20$

Solution:
If $\bar{y}$ denotes the arithmetic of the variable y, then $\bar{y} = 35$ Suppose $x = 3y - 20$

So, $\bar{x} = 3\bar{y} - 20 = 3 \times 35 - 20 = 105$

(d) If there are two groups of values of a variable x, one containing n_1 values with mean $\bar{x}_1$ and other containing n_2 values with mean $\bar{x}_2$, then the mean of the combined data is given by

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Let the values of the first group be $x_{11}, x_{12}, \ldots, x_{1n_1}$, while those in the second group be $x_{21}, x_{22}, \ldots, x_{2n_2}$.

Clearly $\sum_{i=1}^{n_1} x_{1i} = n_1\bar{x}_1$ and $\sum_{i=1}^{n_2} x_{2i} = n_2\bar{x}_2$

Hence sum of all the values in the two groups, taken together, is given by

$$\sum_{i=1}^{n_1} x_{1i} + \sum_{i=1}^{n_2} x_{2i} = n_1\bar{x}_1 + n_2\bar{x}_2$$

Then the mean of the combined data is

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \quad \ldots (3e)$$

Remarks: (i) If there are k groups of values of a variable x such that these groups contain $n_1, n_2, \ldots, n_k$ values and have means $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_k$ respectively, then the combined mean of x is

$$\bar{x} = \frac{\sum_{i=1}^{k} n_i \bar{x}_i}{\sum_{i=1}^{k} n_i} \quad \ldots (4f)$$