If we put $c=\bar{x}$ in (5d), then the measure is referred to as the standard deviation and is denoted by s.

So, the standard deviation may be defined as the positive square root of the mean of the squares of the ~~dev~~ deviations of the variable values from their mean. Thus

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2} \quad \ldots (5e)$$

If the given data are arranged in a frequency table, the standard deviation is given by

$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{r} f_i(x_i-\bar{x})^2} \quad \ldots (5f)$$

Where the symbols have their usual significance.

It should be noted that s^2, the square of the standard deviation, is called the variance of the variable.

To facilitate computation, (5e) may be put in a simpler form. We have

$$\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)$$

$$= \sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + n\bar{x}^2$$

$$= \sum_{i=1}^{n}x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 \quad \left[\because \sum_{i=1}^{n}x_i = n\bar{x}\right]$$

$$= \sum_{i=1}^{n}x_i^2 - n\bar{x}^2$$

Hence,
$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2} \quad \ldots (5g)$$
$$= \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum x_i\right)^2} \quad \ldots (5h)$$

Similarly, on simplification of (5f), we get
$$s = \sqrt{\frac{1}{n}\sum_{i=1}^{r} f_i x_i^2 - \bar{x}^2} \quad \ldots (5i)$$
$$= \sqrt{\frac{1}{n}\sum_{i=1}^{r} f_i x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{r} f_i x_i\right)^2} \quad \ldots (5j)$$

Example 5.6

Calculate the standard deviation of a variable x which takes the values $1, 3, 5, 7, \ldots, 25$.

Solution: Here number of values $(n) = 13$

We calculate
$$\sum_i x_i = 1+3+5+\cdots+25 = \frac{13}{2}(1+25) = 169$$
$$\sum_i x_i^2 = 1^2+3^2+5^2+\cdots+25^2$$
$$= (1^2+2^2+\cdots+26^2) - (2^2+4^2+\cdots+26^2)$$
$$= \frac{26\times 27\times 53}{6} - 2^2(1^2+2^2+\cdots+13^2)$$
$$= 6201 - 4\times\frac{13\times 14\times 27}{6}$$
since $1^2+2^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$
$$= 6201 - 3276 = 2925$$

We have $s = \sqrt{\frac{1}{n}\sum_i x_i^2 - \left(\frac{1}{n}\sum_i x_i\right)^2}$

or, $s = \sqrt{\frac{2925}{13} - \left(\frac{169}{13}\right)^2} = \sqrt{225-169} = \sqrt{56} = 7.48$ (nearly)

Some properties of the standard deviation are as follows:

(i) If all the values of a variable are equal, its standard deviation is zero. The converse is also true.

Proof: Let the variable x assumes n values x_i, $i=1,2,\ldots,n$

and let $x_i = c$ for each i

$$\therefore \bar{x} = \frac{1}{n}\sum_{i=1}^{n} c = c$$

Then $x_i - \bar{x} = 0$, for each i

and $s = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2} = \sqrt{\frac{1}{n}\times 0} = 0$

On the contrary, suppose

$s=0$ or, $s^2=0$ or $\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2 = 0$

or $\sum_{i=1}^{n}(x_i-\bar{x})^2 = 0$, which is possible only when $x_i - \bar{x} = 0$, for each i, i.e., when $x_i = \bar{x}$, for each i.

Thus, if the standard deviation is zero, all the values of the variable are equal.

(ii) If $y = a+bx$ is the relation between two variables x and y, then their respective standard deviations, denoted by s_x and s_y, are related as

$s_y = |b| s_x$ (Prove it)

Example 5.7

Calculate the standard deviation of the frequency distribution given below:

x :	0	1	2	3	4	5	6	7	8
f :	4	10	13	21	23	20	18	9	2

Solution : We prepare the following table for necessary calculations ;

Table 5.4
Necessary Calculation

x	f	fx	fx^2
0	4	0	0
1	10	10	10
2	13	26	52
3	21	63	189
4	23	92	368
5	20	100	500
6	18	108	648
7	9	63	441
8	2	16	128
Total	120	478	2336

We have,

$$s^2 = \frac{1}{n}\sum_i f_i x_i^2 - \left(\frac{1}{n}\sum_i f_i x_i\right)^2 = 19.47 - (3.98)^2$$

$$= 19.47 - 15.84 = 3.63$$

Hence $s_x = \sqrt{3.63} = 1.9$

Example 5.8

A sample of 506 persons showed the income distribution given below :

Income	No. of persons
150-300	232
300-450	128
450-600	60
600-750	40
750-900	28
900-1100	12
1100-1500	6

Find out the standard deviation