(ii) Chi-square (χ^2) Distribution :

A random variable x is said to follow chi-square (χ^2) distribution if its p.d.f is of the form

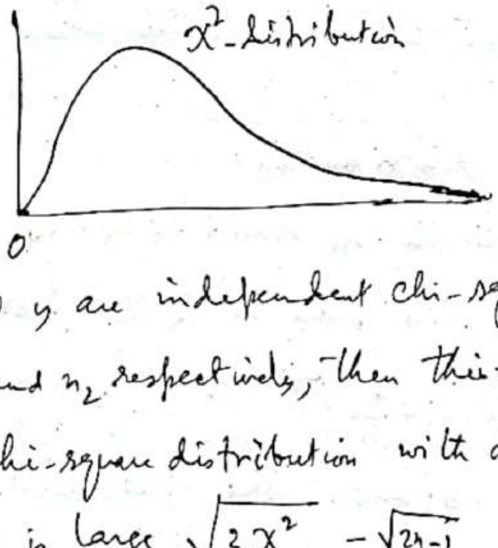
$$f(x) = K \cdot e^{-x/2}\, x^{(n/2)-1} \;;\; (0 < x < \infty),\ K \text{ is constant}$$

The parameter n (positive integer) is called the ~~[illegible]~~ degrees of freedom. Here the random variable x is written as χ^2 (where χ is a letter in Greek alphabet and pronounced as ky) when it follows chi-square distribution.

Characteristics :

1. Mean $= n$, S.d. $= \sqrt{2n}$ where n is the number of degrees of freedom (d.f) of chi-square distribution

2. The Chi-square curve is positively skew, and starting from 0 it extends to infinity in the right (shown in Fig. 1)



3. If x and y are independent chi-square variates with d.f n_1 and n_2 respectively, then their sum $(x+y)$ also follows chi-square distribution with d.f (n_1+n_2).

4. When d.f n is large $\sqrt{2\chi^2} - \sqrt{2n-1}$ approximately follows standard normal distribution

Theorem II : If $z_1, z_2, \ldots, z_n$ are n independent standard normal variates, Then $z_1^2+z_2^2+\cdots+z_n^2 = \sum_{i=1}^{n} z_i^2$

follows chi-square distribution with n degrees of freedom. For this reason, often a χ^2 variate with n d.f is defined to be the sum of square of n independent standard normal variates.

Theorem III : If $x_1, x_2, \ldots, x_n$ is a random sample from a normal population with mean μ and s.d σ then

~~(a) $\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$ follows chi-square distribution with n d.f and~~

~~(b) $\sum_{i=1}^{n} (x_i-\bar{x})^2$ follows chi-square distribution with (n-1) d.f. (Here $\bar{x}$ is the mean of the sample)~~

(a) $\dfrac{\sum_{i=1}^{n} (x_i-\mu)^2}{\sigma^2}$ follows chi-square distribution with n d.f. and

(b) $\dfrac{\sum_{i=1}^{n} (x_i-\bar{x})^2}{\sigma^2}$ follows chi-square distribution with (n-1) d.f where $\bar{x}$ is the mean of the sample.

(iii) Student's t distribution:

A random variable is said to follow Student's t distribution, if its p.d.f is of the form

$$f(t) = K\left(1+\frac{t^2}{n}\right)^{-(n+1)/2} \quad (-\infty < t < \infty)$$

where K is a constant. The parameter n (positive integer) is called the ~~number of~~ degrees of freedom (d.f). The distribution was discovered by W.S. Gossett who wrote under the pen-name "Student" and hence it is called Student's distribution. Also a variable which follows Student's distribution is denoted by the symbol t.

Characteristics: 1. Mean = 0 , s.d = $\sqrt{\frac{n}{n-2}}$; $(n>2)$

2. The t-curve is symmetrical about 0, extending from $-\infty$ to ∞. It has zero skewness and positive kurtosis (leptokurtic), i.e., $\beta_1 = 0$, $\beta_2 > 3$.

3. When the d.f n is large, the t distribution can be approximated by the standard normal distribution.

Theorem IV If z and y are independent random variables, where z follows standard normal distribution and y follows chi-square distribution with n degrees of freedom, then $t = \frac{z}{\sqrt{y/n}}$ follows Student's t distribution with n degrees of freedom.

Theorem V If a random ~~sample~~ sample of size n is drawn from a normal population with mean μ and s.d. σ, then $\frac{\bar{x} - \mu}{S/\sqrt{n-1}}$ follows t distribution with $(n-1)$ degrees of freedom, where $\bar{x}$ and S denote the mean and s.d of the sample.

Theorem VI : If two independent random samples of sizes n_1 and n_2 are drawn from two normal populations with means μ_1 and μ_2 and a common s.d. σ, then

$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{1/n_1 + 1/n_2}}$ follows t distribution with (n_1+n_2-2) degrees of freedom, where $\bar{x}_1$, $\bar{x}_2$ denote the means and S_1, S_2 the s.d.s of the samples and

$$\therefore s = \sqrt{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}}$$

(iv) Snedecor's F distribution:

A random variable is said to follow F distribution with degrees of freedom (n_1, n_2) if its p.d.f is of the form

$$f(F) = K.\, F^{(n_1/2)-1} \left(n_2 + n_1 F\right)^{-(n_1+n_2)/2} \quad 0 < F < \infty$$

where K is a constant. The distribution was discovered by G.W. Snedecor and named F in honour of the distinguished mathematical Statistician Sir R.A. Fisher.

Characteristics: 1. Mean $= \frac{n_2}{n_2 - 2}$ Mode $= \frac{n_2(n_1-2)}{n_1(n_2+2)}$

$s.d = \left(\frac{n_2}{n_2 - 2}\right)\sqrt{\frac{2(n_1+n_2-2)}{n_1(n_2-4)}}$ provided they exist and are positive

2. The F-curve is positively skew and starting from 0 to infinity.

Theorem VII : If y_1, y_2 are independent chi-square variates with degrees of freedom n_1 and n_2 respectively, then

$$F = \frac{y_1/n_1}{y_2/n_2}$$ follows F distribution with d.f (n_1, n_2)

Theorem VIII : If $x_1, x_2, \dots x_{n_1}$ and $y_1, y_2, \dots, y_{n_2}$ are independent random samples of sizes n_1 and n_2 respectively from two normal population with means μ_1, μ_2 and common s.d σ, then

(i) $\frac{\sum(x_i - \mu_1)^2/n_1}{\sum(y_i - \mu_2)^2/n_2}$ follows F distribution with (n_1, n_2) d.f

(ii) $\frac{\sum(x_i - \mu_1)^2/n_1}{\sum(y_i - \bar{y})^2/(n_2-1)}$ follows F distribution with (n_1, n_2-1) d.f

(iii) $\frac{\sum(x_i - \bar{x})^2/(n_1-1)}{\sum(y_i - \mu_2)^2/n_2}$ follows F distribution with (n_1-1, n_2) d.f

(iv) $\frac{\sum(x_i - \bar{x})^2/(n_1-1)}{\sum(y_i - \bar{y})^2/(n_2-1)}$ follows F distribution with (n_1-1, n_2-1) d.f