9. Estimation and Test of ~~Significance~~ Significance

The object of sampling is to study the features of the population on the basis of sample observations. A careful selection of sample is expected to reveal these features, and hence we shall infer about the population from a statistical analysis of the sample. This process is known as Statistical Inference.

There are two types of problems. Firstly, we may have no information at all about some characteristics of the population, especially the values of the parameters involved in the distribution, and it is required to obtain estimates of these parameters. This is the problem of Estimation. Secondly, some information or hypothetical values of the parameters may be available, and it is required to test how far the hypothesis is tenable in the light of the information provided by the sample. This is the problem of Test of Hypothesis or Test of Significance.

9.1 Theory of Estimation : Suppose we have a random sample ~~of~~ $x_1, x_2, \dots, x_n$ on a variable x, whose distribution in the population involves an unknown parameter θ. It is required to find an estimate of θ on the basis of the sample values. The estimation is done in two different ways: (i) Point Estimation (ii) Interval Estimation. In point estimation, the ~~ed~~ estimated value is given by a single quantity, which is a function of sample observations (i.e., statistic). This function is called the 'estimator' of the parameter and the value of the estimator in ~~a~~ a particular sample is called an estimate. In interval estimation, an interval within which the parameter is expected to lie, is given by using two quantities based on sample values. This is known as Confidence Interval and the two quantities which are used to specify the interval, are known as Confidence limits.

9.2 Statistical Hypothesis : In ~~an~~ many practical problems, statisticians are called upon to make decisions about a statistical population on the basis of sample observations. For example, given a random sample, it may be required to decide whether the population, from which the sample has been obtained, is a normal distribution with mean = 40 and s.d = 3. In attempting to reach such decisions, it is necessary to make certain assumptions or guesses about the characteristics of population, particularly about the probability distribution or the value of its parameters. Such an assumption or statement about the population is called a Statistical Hypothesis. The validity of the hypothesis will be tested by analysing the sample. The procedure which enables us to decide whether a certain hypothesis is true or not, is called Test of significance or Test of Hypothesis.

Null hypothesis and Alternative Hypothesis

In tests of signifance, we start with a certain hypothesis about the population charactertistics. This is called Null Hypothesis, and is denoted by the symbol H_0. For example, the null hypothesis may be that the population mean is 40. We write $H_0 \ (\mu = 40)$

Any hypothesis which differs from the null hypothesis is called Alternative Hypothesis, and is denoted by H_1. The null hypothesis is tested against an alternative hypothesis which in the above case, may be that the population mean is not 40 or that it is greater than 40 or that it is less than 40; i.e., any one of

$H_1 (\mu \neq 40)$, $H_1 (\mu > 40)$, $H_1 (\mu < 40)$

The sample is then analysed to decide whether to reject or not to reject the null hypothesis H_0. For this purpose, we choose a suitable statistics, called Test Statistic and find its sampling distribution, assuming that H_0

is really true. The observed value of the statistic in the sample will be in general be different from the expected value because of sampling fluctuations. If the difference between them be large, the null hypothesis H_0 is rejected, and we doubt the validity of our assumption. If the difference is not large, H_0 is not rejected and the difference may be considered to have arisen solely due to fluctuations of sampling. It is therefore necessary to decide how much of difference is tolerable before we are able to conclude that the null hypothesis is acceptable

Level of significance and Critical region

The decision about rejection or otherwise of the null hypothesis is based on probability considerations. Assuming the null hypothesis to be true, we calculate the probability of obtaining a difference equal to or greater than the observed difference. If this probability is found to be small, say less than 0.05, the conclusion is that observed value of the statistic is rather unusual and has arisen because of the underlying assumption (i.e null hypothesis) is not true. We say that the observed difference is significant at 5 percent level and hence the null hypothesis is rejected at 5 percent level of significance. If, however, this probability is not very small, say more than 0.05, the observed difference can not be considered to be unusual and is attributed to sampling fluctuations only. The difference is, now, said to be not significant at 5 percent level ~~of significance~~ and we conclude that there is no reason to reject the null hypothesis at 5 percent level of significance.

Suppose, the sampling distribution of the statistic is a normal distribution. Since the area under the normal curve outside the ordinates at mean $\pm$ 1.96 (s.d) is only 5%, the probability that the observed value of the statistic differs from the expected value 1.96 times the S.E or more is 0.05; and the probability of a larger difference will be still smaller.

If, therefore $z = \frac{\text{observed value} - (\text{Expected value})}{S.E}$ is either greater than 1.96 or less than -1.96 (i.e numerically greater than 1.96, the null hypothesis H_0 is rejected at 5% level of significance

[S.E $\Rightarrow$ The standard deviation calculated from the sampling distribution of a statistic is called its Standard Error or S.E]

The set of values $z \geq 1.96$ or ≤ -1.96, i.e.,

$$|z| \geq 1.96$$

constitutes what is called the Critical region for the test.

Similarly since the area outside the ordinates at mean $\pm$ 2.58 (s.d.) is only 1%, H_0 is rejected at 1% level of significance, if $|z| \geq 2.58$, i.e the critical region is $|z| \geq 2.58$ at 1% level of significance.

Using the sampling distribution of an appropriate test statistic we are thus able to establish the maximum difference at a specified level between the observed and expected values that is consistent with the null hypothesis H_0. The set of values of test statistic corresponding to this difference which lead to the acceptance of H_0 is called Region of acceptance.

Conversely, the set of values of the test statistic leading to the rejection of H_0 is referred to as Region of ~~acceptance~~ rejection or Critical region of the test. The values of the statistic which lies at the boundary of the regions of acceptance and rejection is critical value.

Type I and Type II Errors

The procedure of testing statistical hypothesis does not guarantee that all decisions are perfectly accurate. At times, the tests may lead to erroneous conclusions. That is so, because the decision is taken on the basis of the sample values, which are themselves fluctuating and depend purely on chance.

The errors in statistical decisions are of two types :

(i) Type I error – This is the error committed by the test in rejecting a true null hypothesis

(ii) Type II error – This is the error committed by the test in accepting a false null hypothesis

Considering the previous illustration for testing whether the population mean is 40, i.e., H_0 $(\mu = 40)$, let us imagine that we have a random sample from a population whose mean is really 40. If we apply the test for H_0 $(\mu = 40)$, we might find that the value of test statistic lies in the critical region, thereby leading to the conclusion that the population mean is not 40, i.e., the test rejects the null hypothesis although it is true. We have thus committed what is known as Type I error.

On the other hand, suppose that we have a random sample from a population whose mean is known to be different from 40, say 43. If we apply the test H_0 $(\mu = 40)$, the value of the test statistic may, by chance, lie in the acceptance region, leading to the ~~cost~~ conclusion that mean may be 40, i.e., the test does not reject the null hypothesis H_0 $(\mu = 40)$, although it is false, This is again another form of incorrect decision, and the error thus committed is known as Type II error. We summarise it in a table

Errors in Test of significance H_0 $(\theta = \theta_0)$

True situation	Statistical Decision: $\theta = \theta_0$	$\theta \neq \theta_0$
$\theta = \theta_0$	Correct decision	Type I error
$\theta \neq \theta_0$	Type II error	Correct decision

THIS IS THE END OF MY PORTION OF GE4 NOTES

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