transportation problem given in Example 1

Solution:

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	3	4	4 (1)
O_2	1	4	7	2	$\boxed{6}$ 1	8 (0)
O_3	3	9	4	8	12	9 (1)
	3	3	4	5	6	
	(1)	(5)	(3)	(1)	(6)	

Maximum penalty is 6 in the 5th column. The minimum cost in this column is $c_{25} = 1$. So, we take

$$x_{25} = \min(a_2, b_5) = \min(8, 6) = 6$$

Since the demand of the column 5 is satisfied, we cross out the 5th column. By adjusting the demand and supply, we go to the next table

	D_1	D_2	D_3	D_4	
O_1	2	11	10	3	4 (1)
O_2	1	$\boxed{2}$ 4	7	2	2 (1)
O_3	3	9	4	8	9 (1)
	3	3	4	5	
	(1)	(5)	(3)	(1)	

In the same way, $x_{22} = 2$, we cross out row 2. The next table is

	D_1	D_2	D_3	D_4	
O_1	2	11	10	3	4 (1)
O_3	3	9	$\boxed{4}$ 4	8	9 (1)
	3	1	4	5	
	(1)	(2)	(6)	(5)	

So, $x_{33} = 4$ and we cross out D_3 (column 3). The next table is

	D_1	D_2	D_4	
O_1	2	11	3	4 (1)
O_3	$\boxed{3}$ 3	9	8	5 (5)
	3	1	5	
	(1)	(2)	(5)	

So, we can choose either row 3 or column 1, Here we take row 3, and the cell with minimum cost is (3, 1). So, $x_{31} = 3$ and we cross out column 1. The next table is

	D_2	D_4	
O_1	11	$\boxed{4}$ 3	4 (8)
O_3	9	8	2 (1)
	1	5	
	(2)	(5)	

So, $x_{14} = 4$, and we cross out row 1. The next table is

	D_2	D_4	
O_3	$\boxed{1}$ 9	$\boxed{1}$ 8	2
	1	1	

So, $x_{32} = 1$, $x_{34} = 1$

So, an initial basic feasible solution is shown in the following table:

	D_1	D_2	D_3	D_4	D_5	
O_1	2	11	10	$\boxed{4}$ 3	7	4
O_2	1	$\boxed{2}$ 4	7	2	$\boxed{6}$ 1	8
O_3	$\boxed{3}$ 3	$\boxed{1}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	12	9
	3	3	4	5	6	

The whole computation can be shown in a single table in the following manner:

	D_1	D_2	D_3	D_4	D_5						
O_1	2	11	10	$\boxed{4}$ 3	7	4 (1)	4 (1)	4 (1)	4 (1)	4 (8)	
O_2	1	$\boxed{2}$ 4	7	2	$\boxed{6}$ 1	8 (0)	2 (1)				
O_3	$\boxed{3}$ 3	9	$\boxed{4}$ 4	8	12	9 (1)	9 (1)	9 (1)	5 (5)	2 (1)	2
	3 (1)	3 (5)	4 (3)	5 (1)	6 (6)						
	3 (1)	3 (5)	4 (3)	5 (1)							
	3 (1)	1 (2)	4 (6)	5 (5)							
	3 (1)	1 (2)		5 (5)							
		1 (2)		5 (5)							
		1		1							

number of occupied cell is $7 = (5+3-1)$ and so This gives a non-degenerate basic feasible solution.

So, a basic feasible solution by VAM is

$$x_{14} = 4,\ x_{22} = 2,\ x_{25} = 6,\ x_{31} = 3,\ x_{32} = 1,\ x_{33} = 4,\ x_{34} = 1$$

and the corresponding Cost $= 4\times3 + 2\times4 + 6\times1 + 3\times3 + 1\times9 + 4\times4 + 1\times8$
$= 68$

Note: we see that, VAM gives the best initial basic feasible solution in respect of cost as we have used the same problem, in calculating initial basic feasible solution in the above examples.

Towards optimality

~~How to check optimality of a basic feasible solution?~~

Consider a balanced transportation problem with m origins and n destinations. The availabilities are a_i, $i=1,2,\dots,m$ and demands are b_j, $j=1,2,\dots,n$ and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Step 1 we first find a basic feasible solution of the given balanced transportation problem by any one of the methods discussed earlier (The best method for finding initial basic feasible solution is VAM). Initial

~~Non-degenerate case~~ ~~Basic~~

basic ~~components~~ feasible components are shown in the upper left corner of the cell. There will be basic $(m+n-1)$ cells for non-degenerate ~~cells~~ solutions. For degenerate solution there will be less than $(m+n-1)$ basic cell. So, we have to choose that number of cells by which the basic component is less ~~and~~ in such a way that together with the basic cells, these $(m+n-1)$ cells does not contain any loop. So, ~~those $(m+n-1)$ cell~~ if choose the basic component as zero for those cells then those $(m+n-1)$ cells gives you a degenerate basic feasible solution. Now for those cells in the basic component where zero is taken for the variable, put $\epsilon > 0$ (a very small number) for ~~I~~ zero and then proceed to the next for both non-degenerate and degenerate solution.

Step 2 Determine a set of $(m+n)$ number u_i and v_j, $i=1,2,\dots,m$, $j=1,2,\dots,n$ such that for all occupied (basic) cells, $c_{ij} = u_i + v_j$.

In practice, to find u_i and v_j, put any one of them