equal to zero and then considering the relation $c_{ij} = u_i + v_j$ for all occupied cells, all other u_i's and v_j's can be found out. Generally, that u_i or v_j for which the corresponding row or column contains maximum number of occupied cells is put to zero.

Step 3 Calculate the cell evaluation Δ_{ij} for each unoccupied cell by the formula $\Delta_{ij} = c_{ij} - (u_i + v_j)$ and put it in the middle of that cell within a circle.

Case 1 If all $\Delta_{ij} \geq 0$ then the solution obtained is optimal. If all $\Delta_{ij} > 0$, then optimal solution is unique but if $\exists$ at least one $\Delta_{ij} = 0$ then it is not unique.

Case 2 If at least on $\Delta_{ij} < 0$, then the solution is not optimal and we go to step 4.

Step 4 Now to find a new basic feasible solution, we choose that (r, s) cell for which the net evaluation $\Delta_{rs} = c_{rs} - (u_r + v_s)$ is most minimum of all those Δ_{ij} for which $\Delta_{ij} < 0$

Form a loop connecting this entering (r, s) cell and those basic cells which will be needed for this. Such loop always exists. Starting from this (r, s) cells, allocate an amount θ with alternative positive and negative signs to all the end points of the closed loop so that supply and demand constraints are always satisfied. The value of θ is chosen as the minimum quantity which will render non-negative values for all the basic variables in the new solution. This value of θ can be easily seen from the allocations made in the cells of the loop. That cell where evaluation will become zero will be considered as leaving basic variable cell (or basic cell).

Step 5 Repeat the steps 2 to step 4 until all the evaluations to the empty cells are negative.

Example 5: Obtain an optimum basic feasible solution to the following transportation problem:

	D_1	D_2	D_3	D_4	Availability
O_1	19	14	23	11	11
O_2	15	16	12	21	13
O_3	30	25	16	39	18
Requirements	6	10	11	15	

Solution: In this problem $\sum a_i = \sum b_j = 40$, so it is a balanced Transportation problem.

We shall first determine an initial basic feasible solution to this problem using VAM as shown in the Table 1

Table 1

	D_1	D_2	D_3	D_4	a_i				
O_1	19	14	23	$\boxed{11}$ 11	11 (3)				
O_2	$\boxed{6}$ 15	$\boxed{3}$ 16	12	$\boxed{4}$ 21	13 (3)	13 (3)	9 (3)	3 (4)	3
O_3	30	$\boxed{7}$ 25	$\boxed{11}$ 16	39	18 (9)	18 (9)	18 (9)	18 (9)	7
b_j	6	10	11	15					
	(4)	(2)	(4)	(10)					
	6	10	11	4					
	(15)	(9)	(4)	(18)					
	6	10	11						
	(15)	(9)	(4)						
		10	11						
		(9)	(4)						
		10							

Number of basic cells is 6 (= 4+3-1) and so a non-degenerate basic feasible solution is given by these cells. The allocations in the basic cell are shown in the upper left corner of each basic cell.

The initial basic feasible solutions as calculated by VAM is shown separately in the Table 2 and uv-method is used to calculate the optimal solution

Table 2

	D_1	D_2	D_3	D_4	u_i
O_1	(14) 19	(8) 14	(26) 23	$\boxed{11}$ 11	-10
O_2	$\boxed{6}$ 15	$\boxed{3}$ 16	(5) 12	$\boxed{4}$ 21	0
O_3	(6) 30	$\boxed{7}$ 25	$\boxed{11}$ 16	(9) 39	9
v_j	15	16	7	21	

Calculations of u_i and v_j :
Let us assume that $u_2 = 0$ (since 2nd row contains maximum number of basic cells).

For basic cell, $c_{ij} = u_i + v_j$

Basic variables	(u,v) equations	Solutions
x_{14}	$u_1 + v_4 = 11 \Rightarrow$	$u_1 = -10$
x_{21}	$u_2 + v_1 = 15 \Rightarrow$	$v_1 = 15$
x_{22}	$u_2 + v_2 = 16 \Rightarrow$	$v_2 = 16$
x_{24}	$u_2 + v_4 = 21 \Rightarrow$	$v_4 = 21$
x_{32}	$u_3 + v_2 = 25 \Rightarrow$	$u_3 = 9$
x_{33}	$u_3 + v_3 = 16 \Rightarrow$	$v_3 = 7$

Cell evaluation for non basic cells $\Delta_{ij} = c_{ij} - (u_i + v_j)$

Non-basic variables	Δ_{ij}
x_{11}	$\Delta_{11} = c_{11} - (u_1 + v_1) = 19 - (-10 + 15) = 14$
x_{12}	$\Delta_{12} = c_{12} - (u_1 + v_2) = 14 - (-10 + 16) = 8$
x_{13}	$\Delta_{13} = c_{13} - (u_1 + v_3) = 23 - (-10 + 7) = 26$
x_{23}	$\Delta_{23} = c_{23} - (u_2 + v_3) = 12 - (0 + 7) = 5$
x_{31}	$\Delta_{31} = c_{31} - (u_3 + v_1) = 30 - (9 + 15) = 6$
x_{34}	$\Delta_{34} = c_{34} - (u_3 + v_4) = 39 - (9 + 21) = 9$

Since $\Delta_{ij} \geqslant 0$ for all non-basic cells, so the optimality condition is satisfied.

So, $x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$ and $x_{33} = 11$ is an optimal solution and the minimum cost is

$$= 11 \times 11 + 6 \times 15 + 3 \times 16 + 4 \times 21 + 7 \times 25 + 11 \times 16$$
$$= 121 + 90 + 48 + 84 + 175 + 176$$
$$= 694$$