	W_1	W_2	W_3	W_4	u_i
F_1	$\boxed{20}$ 3	8	$\boxed{10}$ 7	(1) 4	1
F_2	(4) 5	$\boxed{50}$ 2	(4) 9	(4) 5	−1
F_3	(2) 4	$\boxed{10}$ 3	$\boxed{30}$ 6	$\boxed{40}$ 2	0
F_4	(4) 0	(3) 0	$\boxed{15}$ 0	(4) 0	−6
v_j	2	3	6	2	

So, $\Delta_{ij} > 0$ for all non-basic cells, so the optimality condition is satisfied

So an optimal solution is

$x_{11} = 20$, $x_{13} = 10$, $x_{22} = 50$, $x_{32} = 10$, $x_{33} = 30$, $x_{34} = 40$, $x_{43} = 15$

and minimum cost $= 20\times3 + 10\times7 + 50\times2 + 10\times3 + 30\times6 + 40\times2 + 15\times0$

$= 520$

Since there is no factory F_4, so $x_{43} = 15$ is of no use; so idle capacity of the warehouse W_3 is 15.

Exercises 1. Solve the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	19	14	23	11	11
O_2	15	16	12	21	13
O_3	30	25	16	39	18
b_j	6	10	11	15	

2. Find an optimal basic feasible solution and minimum cost to the following transportation problem:

	D_1	D_2	D_3	
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
	50	80	80	

3. Solve the following transportation :

To

				Available
From	2	7	4	5
	3	3	1	8
	5	4	7	7
	1	6	2	14
Required	7	9	18	

4. A company has three plants at locations A, B and C, which supply to warehouses located at D, E, F, G and H. Monthly plant capacities are 800, 500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 500 units respectively. Unit transportation costs (in rupees) are given below

To

		D	E	F	G	H
From	A	5	8	6	6	3
	B	4	7	7	6	5
	C	8	4	6	6	4

Determine an optimum distribution for the company in order to minimize the total transportation cost.

ASSIGNMENT PROBLEM

A special type of transportation problem is an assignment problem. In this problem, a number of operations are to be assigned to an equal number of operators, where each operator will perform only one operation. In practical field, such problems are to assign men to offices, jobs to machines, jobs to workers etc. and the object of this problem is to minimize the overall cost for given assignment schedule. An assignment problem is always a degenerate problem. The units available at each origin and units demanded at each destination are all equal to 1. In an $n \times n$ assignment problem, number of occupied cells is n instead of $n+n-1 (= 2n-1)$ and so it is always a degenerate problem. The cost matrix for the problem is as follows:

Workers

$$
\begin{array}{c|cccccc|c}
 & 1 & 2 & 3 & \cdots & \cdots & n & a_i \\
\hline
1 & c_{11} & c_{12} & c_{13} & \cdots & \cdots & c_{1n} & 1 \\
2 & c_{21} & c_{22} & c_{23} & \cdots & \cdots & c_{2n} & 1 \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\
\vdots & \cdots & & \cdots & \cdots & \cdots & \cdots & \vdots \\
n & c_{n1} & c_{n2} & c_{n3} & \cdots & \cdots & c_{nn} & 1 \\
\hline
b_j & 1 & 1 & 1 & \cdots & \cdots & 1 &
\end{array}
$$

Jobs

Mathematical formulation of assignment problem:

Let c_{ij} denotes the cost of assigning ith job to the jth worker and $x_{ij} = \begin{cases} 1 & \text{if the } i\text{th worker is assigned to the } j\text{th job} \\ 0, & \text{otherwise} \end{cases}$. Then the assignment problem can be mathematically written as follows:

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n$$

Since x_{ij} can assume only the values 0 and 1 in assignment problem, it can not ~~to~~ be treated as a linear programming problem. Again the constraints indicate that only one job is assigned to one worker and only one worker should be assigned with one job.

Solution of an assignment problem: Formation of an assignment problem indicates that it is a special transportation problem in which the number of occupied cells are n instead of $n+n-1 = 2n-1$. So, any of its basic feasible solution will contain $(2n-1)$ variables of which n variables are each equal to 1 and other $(n-1)$ variables are equal to zero. Hence, there is a high level of degeneracy in an assignment problem. If this problem is solved by the method as applied to solve the transportation problem, we see that we have to perform a large number of iterations to get an optimal ~~solution~~ solution. Due to this disadvantage, an assignment problem is solved by a separate method, called Hungarian method which is based on the following two theorems:

Theorem 1 If a constant be added to (or subtracted from) every element of any row and/or any column of the cost matrix of an assignment ~~probjec~~ problem, the resulting assignment problem has the same optimal solution as the original problem.

Proof Let in an assignment problem, we have the cost matrix $C = (c_{ij})_{n \times n}$ $i=1,2,\dots n,\ j=1,\dots n$ and let .