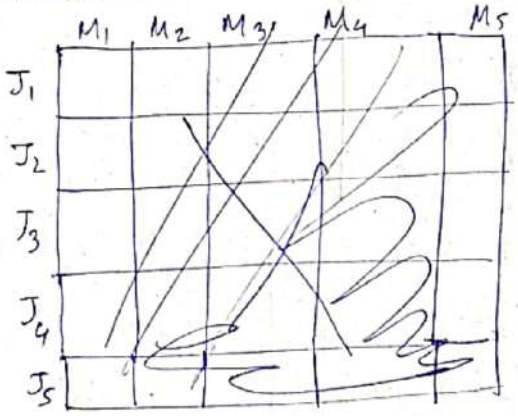
Unbalanced assignment problem :

If in any assignment problem the number of jobs and the number of workers are not equal, it is known as unbalanced assignment problem. This unbalanced assignment problem can be converted to a balanced problem just adding a fictitious job or worker, whichever has the deficiency, with zero cost in the corresponding row or column in the cost matrix. Then assignment algorithm is applied to this balanced problem.

Example 1 Find the optimal assignment for the following assignment problem and the minimum cost:



	M_1	M_2	M_3	M_4	M_5
J_1	3	8	2	10	3
J_2	8	7	2	9	7
J_3	6	4	2	7	5
J_4	8	4	2	3	5
J_5	9	10	6	9	10

Solution : This is a balanced assignment problem

We first subtract the minimum cost of the row from every element of that row and do it for all rows.

Table 1

	M_1	M_2	M_3	M_4	M_5
J_1	1	6	0	8	1
J_2	6	5	0	7	5
J_3	4	2	0	5	3
J_4	6	2	0	1	3
J_5	3	4	0	3	4

Next, we subtract the minimum element of the column from to all the elements of the column and do it for all columns

Table 2

	M_1	M_2	M_3	M_4	M_5
J_1	0	4	0	7	0
J_2	5	3	0	6	4
J_3	3	0	0	4	2
J_4	5	0	0	0	2
J_5	2	2	0	2	3

Now we draw minimum of horizontal and vertical lines to cover all the zeros.

Table 3

	M_1	M_2	M_3	M_4	M_5
J_1	0	4	0	7	0
J_2	5	3	0	6	4
J_3	3	0	0	4	2
J_4	5	0	0	0	2
J_5	2	2	0	2	3

As the number of lines covering zeros $= 4 < 5$, ~~so we go to~~ So, ~~the next step~~ optimal condition is not reached.

The smallest uncovered element of Table 3 is 2. Subtracting 2 from each uncovered element adding 2 to the elements at the intersection of two lines, we get the

we get the new matrix as given in Table 4

Table 4

	M_1	M_2	M_3	M_4	M_5
J_1	0	4	2	7	0
J_2	3	1	0	4	2
J_3	3	0	2	4	2
J_4	5	0	2	0	2
J_5	0	0	0	0	1

As the number of lines covering zeros = 5 = order of the matrix, so the optimal condition is satisfied. So, we make the optimal assignment in the Table 5 as follows:

	M_1	M_2	M_3	M_4	M_5
J_1	⊗				[0]
J_2			[0]		
J_3		[0]			
J_4		⊗		[0]	
J_5	[0]	⊗	⊗	⊗	

So, the optimal assignment is $J_1 \to M_5$

$J_2 \to M_3$

$J_3 \to M_2$

$J_4 \to M_4$

$J_5 \to M_1$

And the minimum cost is $= 3+2+4+3+9 = 21$

Example 2 A company has four machines on which one to do four jobs. Each job can be assigned to one

and only machine. The cost of each job on each machine is given in the following table:

Machines

Jobs	I	II	III	IV
A	42	35	28	21
B	30	25	20	15
C	35	25	20	15
D	24	20	16	12

Find the optimal cost assignment and the minimum cost.

Solution: This is a balanced transportation problem. We first subtract the minimum element of the row from each element of that row and do it for all rows.

Table 1

	I	II	III	IV
A	21	14	7	0
B	15	10	5	0
C	15	10	5	0
D	12	8	4	0

Next, we subtract the minimum element of the column from each element of that column and do it for all columns.

Table 2

	I	II	III	IV
A	9	6	3	0
B	3	2	1	0
C	3	2	1	0
D	0	0	0	0

Now we draw minimum no. of horizontal and vertical lines to cover all the zeros.