Table 3

	I	II	III	IV
A	9	6	3	0
B	3	2	1	0
C	3	2	1	0
D	0	0	0	0

As minimum of lines covering all the zeros are $2 < 4 =$ order of the matrix, So, optimal condition is not satisfied.

The smallest uncovered element in table 3 is 1. Subtracting 1 from each uncovered element and adding 1 to the elements at the intersection of two lines, we get a new matrix in Table 4

Table 4

	I	II	III	IV
A	8	5	2	0
B	2	1	0	0
C	2	1	0	0
D	0	0	0	1

As the minimum of lines covering zeros are $3 < 4$, so optimality is not reached.

The smallest uncovered element in Table 4 is 1. Subtracting 1 from each uncovered elements and addition 1 to the elements at the intersection of two lines, we get table 5

Table 5

	I	II	III	IV
A	7	6	2	0
B	1	0	0	0
C	1	0	0	0
D	0	0	1	2

So, the minimum number of lines covering zeros are $4 =$ order of the matrix, so optimal condition is satisfied.

Now we make the optimal assignment

Table 6

	I	II	III	IV
A				$\boxed{0}$
B		$\boxed{0}$	X	X
C		X	$\boxed{0}$	X
D	$\boxed{0}$	X		

	I	II	III	IV
A				$\boxed{0}$
B		X	$\boxed{0}$	X
C		$\boxed{0}$	X	X
D	$\boxed{0}$	X		

So, the optimal assignments are (not unique)

(i)	(ii)
$A \to IV$	$A \to IV$
$B \to III$	$B \to II$
$C \to II$	$C \to III$
$D \to I$	$D \to I$

and minimum cost $= 21 + 20 + 25 + 24$
$= 21 + 25 + 20 + 24 = 90$

Example 3 Solve the assignment problem given by the following cost matrix:

	I	II	III	IV
A	18	17	12	11
B	19	15	11	16
C	25	21	17	11
D	16	14	11	11

Solution: This is a balanced problem. ~~and we solve~~

Subtracting the minimum cost of each row from every element of the same row, we get the corresponding matrix given in Table 1:

Table 1

7	6	1	0
8	4	0	5
14	10	6	0
5	3	0	0

Then we subtract the minimum cost of each column from every element of the same column and we get the revised cost matrix given in Table 2:

Table 2

2	3	1	0
3	1	0	5
9	7	6	0
0	0	0	0

Minimum number of lines (horizontal & vertical) to cover all the zeros of table 2 is $3 < 4$. So, optimality is not reached and we got to the next table.

The smallest uncovered element in Table 2 is 1. Now, subtracting 1 from each uncovered elements and adding 1 to the elements at the intersection of two lines, we get a new cost matrix given in Table 3 :

1	2	0	0
3	1	0	6
8	6	5	0
0	0	0	1

Table 3

1	2	0	0
3	1	0	6
8	6	5	0
0	0	0	1

Again the minimum number of horizontal and vertical lines covering all zeros in Table 3 is $3 < 4$. So, optimal condition is not satisfied and we go to the next table

The smallest uncovered element in Table 3 is 1, Now subtracting 1 from each uncovered element and adding 1 to the elements at the intersection of two lines, we get a new cost matrix given in Table 4

Table 4

0	1	0	0
2	0	0	6
7	5	5	0
0	0	1	2

Now we make the optimal assignment

	I	II	III	IV
A	$\cancel{0}$		$\boxed{0}$	$\cancel{0}$
B		$\boxed{0}$	$\cancel{0}$	
C				$\boxed{0}$
D	$\boxed{0}$	$\cancel{0}$		

	I	II	III	IV
A	$\boxed{0}$		$\cancel{0}$	$\cancel{0}$
B		$\cancel{0}$	$\boxed{0}$	
C				$\boxed{0}$
D	$\cancel{0}$	$\boxed{0}$		

So, the optimal assignments are :

(i) $A \to III$, $B \to II$, $C \to IV$, $D \to I$

and (ii) $A \to I$, $B \to III$, $C \to IV$, $D \to II$

And the minimum cost is $= 12+15+11+16$ (for (i))

$= 18+11+11+14$ (for (ii))

$= 54$

Exercises 1. Find the optimal assignment and the minimum cost for the assignment problems with the following cost matrices :

(i)

	J_1	J_2	J_3
P_1	12	24	15
P_2	23	18	24
P_3	30	19	28

(ii)

	1	2	3	4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

(iii)

	I	II	III	IV
A	10	12	19	11
B	5	10	7	8
C	6	4	5	7
D	5	7	7	6

2. Solve the following assignment problems :

(i)

	M_1	M_2	M_3	M_4	M_5
J_1	8	4	2	6	1
J_2	0	9	5	5	4
J_3	3	8	9	2	6
J_4	4	3	1	0	3
J_5	9	5	8	9	5

(ii)

	1	2	3	4
A	5	3	1	8
B	7	9	2	6
C	6	4	5	7
D	5	7	7	6

This is the end of GE3 Notes