Note In practical problems, the constraints are generally so expressed that $b_i \geq 0$, $i=1,2,\ldots,m$. and after that ~~slack~~ slack and surplus variables are introduced.

Formulation of LPP

Here are some examples of practical problems which can be formulated mathematically as an LPP

Example 1 A firm can produce three types of cloth, say A, B and C. Three kinds of wool are required for it, say red, green and blue wool. One unit length of type A cloth needs 2 yards of red wool and 3 yards of blue wool. One unit length of type B cloth needs 3 yards of red wool, 2 yards of green wool and 2 yards of blue wool. One unit length of type C cloth needs 5 yards of green wool and 4 yards of blue wool. The firm has only a stock of 8000 yards of red wool, 10000 yards of green wool and 15000 yards of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs 5, of type B is Rs 7 and type C ~~cloth~~ cloth is Rs. 6. Formulate this as an LPP so as to maximize the income from the finished cloth.

Solution: Let the firm ~~of~~ produces x_1 units of type A cloth, x_2 units of type B cloth and x_3 units type C ~~cloth~~ cloth.

The problem can be represented in a tabular form in the following way:

Quality of wool	Quantities (in yards) of the wools required for one unit of the cloth of type: A	B	C	Total quantity of available wool (in yards)
Red	2	3	0	8000
Green	0	2	5	10000
Blue	3	2	4	15000

So, from the table, we get the constraints as follows:

$$2x_1 + 3x_2 \leq 8000 \quad \text{(for red wool)}$$
$$2x_2 + 5x_3 \leq 10000 \quad \text{(for green wool)}$$
$$3x_1 + 2x_2 + 4x_3 \leq 15000 \quad \text{(for blue wool)}$$

Also the firm can not produce any negative quantities, so, $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$

Again total income from three types of cloth is

$$Z = 5x_1 + 7x_2 + 6x_3$$

So, the problem can be written as the following LPP:

Maximize $Z = 5x_1 + 7x_2 + 6x_3$

subject to

$$2x_1 + 3x_2 \leq 8000$$
$$2x_2 + 5x_3 \leq 10000$$
$$3x_1 + 2x_2 + 4x_3 \leq 15000$$

and $x_1, x_2, x_3 \geq 0$

Example 2

At a cattle breeding firm it is prescribed that the food ration for one contains at least 12, 22 and 2 units of nutrients A, B and C respectively. Two different kinds of fodder are available in the market. Each gram of these two fodders contains the following amounts of the three nutrients

	Fodder 1	Fodder 2
Nutrient A	2	3
Nutrient B	1	1
Nutrient C	2	1

It is given that the costs of per gram of fodder 1 and fodder 2 are Rs. 20 and Rs 30 respectively. Pose this as an LPP in terms of minimizing the cost of purchasing the fodders for the above cattle breeding firm.

Solution: Let x_1 gram and x_2 gram of fodder 1 and 2 respectively be used for ration for each animal to satisfy the requirement of minimum nutrients. Then the total cost of purchase is $20x_1 + 30x_2$ in Rs. Also the total amount of nutrients A, B and C are $2x_1+3x_2$, x_1+x_2 and $2x_1+x_2$ respectively. So the problem can be written as the following LPP:

$$\text{Minimize } Z = 20x_1 + 30x_2$$

$$\text{subject } \quad 2x_1 + 3x_2 \geq 12$$

$$x_1 + x_2 \geq 22$$

$$2x_1 + x_2 \geq 2$$

$$\text{and } \quad x_1, x_2 \geq 0$$

Example 3 The daily requirement of a patient of vitamin A and vitamin B are at least 80 units and 100 units respectively. Two foods X and Y are available in the market which contain vitamin A and vitamin B. Food X contains 2 units of vitamin A and 3 units of vitamin B per gram. Food Y contains 4 units of vitamin A and 5 units of vitamin B per gram. Formulate the above problem as an LPP so as to minimize the cost.

Solution: The problem may be tabulated as follows:

Types of food	Units of Vitamin present per gram.		Cost per gram of the foods (in Rs.)
	A	B	
X	2	3	1
Y	4	5	2

Let x grams of food X and y grams of food Y be purchased. So, the total cost is $x+2y$ in Rs. In the total food, the amount of Vitamin A present is $2x+4y$, and the amount of vitamin B present is $3x+5y$

So, the LPP becomes

Minimize $z = x+2y$

subject to $2x+4y \geq 80$

$3x+5y \geq 100$

and $x, y \geq 0$

Example 4 A manufacturer produces two types of commodities. Production cost of one of the each type of commodities are Rs. 100 and Rs. 150 respectively and times needed are 6 hours and 10 hours respectively. He can work 12 hours per day and his capital is Rs. 10000. The profit on each type of commodities are Rs. 10 and Rs. 15 respectively. Pose this problem as an LPP so that his profit per week is maximum

Solution: The problem can be expressed in tabular form as follows:

Types of Commodities	Production cost for each in Rs.	Time needed for each in hours	Profit for each in Rs.	Total capital in Rs.	Total working hours in a week
I	100	6	10	10000	7×12 = 84
II	150	10	15		

Let the number of commodities produced be x_1 of Type I and x_2 of Type II per week. Now the total production cost in Rs is $100x_1 + 150x_2$ and total working hour needed $6x_1 + 10x_2$. The total profit is $10x_1 + 15x_2$

So, the LPP takes the form:

Maximize $Z = 10x_1 + 15x_2$

subject to $100x_1 + 150x_2 \leq 10000$

$6x_1 + 10x_2 \leq 84$

and $x_1, x_2 \geq 0$