Exercises: 1. Mr. X requires at least 10, 12 and 12 units of chemicals A, B and C respectively for his garden. One liquid product contains 5, 2 and 1 units of A, B and C respectively. A dry product contains 1, 2 and 4 units of A, B and C respectively. If the liquid product ~~set~~ sells for Rs 30 and the dry product sells for Rs. 20, pose a linear programming problem so as to minimize the cost.

2. A person has two types of machines and he must have at least 2 first type of machine and 5 second type of machine. The cost of each first type machine is Rs. 2000 and it requires 20 square ~~metres~~ metres space, whereas the cost of each second type machine is Rs. 1500 and it requires 30 square metres space. His capital is Rs. 20000. and the available space is ~~200~~ 220 square metres. Profit from each first type of machine is Rs. 70 and that from each second type machine is Rs. 110. Formulate this as an LPP so as to maximize the profit earned.

3. A tailor has 80 square metre of cotton material and 120 square metre of woolen material. A suit requires 1 square metre of cotton and 3 square metre of woolen material and a dress requires 2 square metre of each. Profit from a suit is Rs 120 and from a dress is Rs. 150. Pose this as an LPP so as to maximize the profit.

Standard form of an LPP and its matrix form

When slack and surplus variables are introduced in the constraints, we introduce those variables in the objective function with zero coefficients. So, after the introduction of slack and surplus variables, the LPP takes the form

Optimize (Maximize or Minimize)

$$\left.\begin{array}{l} Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} \quad a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ \qquad a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \qquad \cdots \quad \cdots \quad \cdots \quad \cdots \\ \qquad a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{array}\right\} \cdots (1)$$

where $x_1, x_2, \ldots, x_n \geq 0$

This form is called the standard form of an LPP.

In matrix notation, the standard form (1) of an LPP can be written as

Optimize (Maximize or Minimize)

$$Z = cx$$

subject $Ax = b$

and $x \geq 0$

Where $c = (c_1, c_2, \ldots c_n)$, $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$, $A = [a_{ij}]_{m \times n}$

$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$

<u>Feasible solution and Optimal solution</u>:

A set of values of decision variables $x_1, x_2, \ldots, x_n$

which satisfies the set of constraints and the non-negativity restrictions is called a feasible solution for the general LPP. That feasible solution of an LPP which will optimize (maximize or minimize) the objective function is called an optimal solution for that LPP.

Note: In an LPP, a maximization problem can be transformed to minimization problem and a minimization problem can be transformed to maximization problem as

$$\text{Max } Z = -\text{Min}(-Z)$$

and $$\text{Min } Z = -\text{Max}(-Z)$$

Worked out examples

1. Introduce slack and surplus variables, convert the LPP to standard form and also write down the matrix form.

(i) Maximize $Z = 2x_1 + 3x_2$

Subject to $x_1 + 2x_2 \leq 5$

$-3x_1 + 5x_2 \geq 8$

$x_1, x_2 \geq 0$

(ii) Minimize $Z = -3x_1 + 2x_2$

Subject to $x_1 - 4x_2 \leq -14$

$-3x_1 + 2x_2 \leq 6$

$x_1, x_2 \geq 0$

Solution: (i) Introducing a slack variable x_3 and a surplus variable x_4, the problem can be written in the standard form as follows:

Maximize $Z = 2x_1 + 3x_2 + 0.x_3 + 0.x_4$

subject to $x_1 + 2x_2 + x_3 = 5$

$-3x_1 + 5x_2 - x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

and the matrix of the LPP is

Maximize $Z = cx$

subject to $Ax = b$

$x \geq 0$

when $c = (2, 3, 0, 0)$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ $b = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -3 & 5 & 0 & -1 \end{pmatrix}$, $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

(ii) First we convert the constraints with requirement parameters are always positive.

So, $x_1 - 4x_2 \leq -14$

becomes $-x_1 + 4x_2 \geq 14$

So, introducing surplus variable x_3 and slack variables x_4, the standard form of the LPP is

Minimize $Z = -3x_1 + 2x_2 + 0.x_3 + 0.x_4$

subject to $-x_1 + 4x_2 - x_3 = 14$

$-3x_1 + 2x_2 + x_4 = 6$

$x_1, x_2, x_3, x_4 \geq 0$