Solution of LPP by graphical method (for two variables)

In an LPP when the decision variables are only two, The simplest method of solving this problem is Graphical method. In This method, at first, in a two dimensional plane, two rectangular coordinate axes are chosen. Then considering the inequation of the constraints as of the problem as equation, the lines represented by them are drawn in the graph and also the non-negativity restrictions are used. Then considering the nature of the constraints (inequations or equations) a zone is marked (if possible) in two dimensional coordinate plane in the first quadrant where all the given restrictions are fulfilled. The permissible Zone satisfied by the decision variables is called feasible Zone or feasible region. Now the problem reduces to find that point (or points) in the feasible region, which will optimize (i.e., maximize or minimize) the objective function (which is linear in this case)

The problem is solved by either of the following approaches:

(a) The values of the decision variables are calculated at each corner of the feasible region and the corresponding values of the ~~of~~ objective function are also calculated. Then, that corner is selected for which the objective function has its optimal (i.e., maximum or minimum) value and the values of the decision variables at that corner are taken as the solution of the problem.

(b) Let the objective function in two variables be of the form $z = c_1x_1 + c_2x_2$, where x_1 and x_2 be two decision variables and c_1 and c_2 are price parameters.

Now the problem reduces to find the point or points in the feasible region which will optimize (i.e., maximize or minimize) the objective function z. By giving any particular value of z, a line is obtained and on this line the objective function will have the same value. Now giving different values of z, the line is moved parallely and ultimately if this line passes through only one point of the feasible region then the optimal value of the objective function is found at that point. If this point is nearest to the origin ~~then~~ then this value is minimum whereas if this point is farthest from the origin, it will be maximum.

It may also be the case that such a line will coincide with one of the edges of the feasible region and in that case every point on that edge gives the optimal value of z and the problem has an infinite number of solutions.

The solution of an LPP may be generally of the following natures:

(i) The feasible region is bounded and the problem has a unique solution;

(ii) The feasible region is bounded and the problem has infinite number of solutions;

(iii) The feasible region is unbounded and the problem has a unique solution;

(iv) The feasible region is unbounded and the problem has an infinite number of solutions;

(v) The feasible region is unbounded and the problem has no solution. and

(vi) No feasible region and so the problem has no solution.

Example 1 ~~(a)~~ Draw graphically the feasible space of the following LPP and solve it:

(a) Maximize $z = 2x_1 + x_2$

subject to $4x_1 + 3x_2 \le 12$

$4x_1 + x_2 \le 8$

$x_1, x_2 \geq 0$

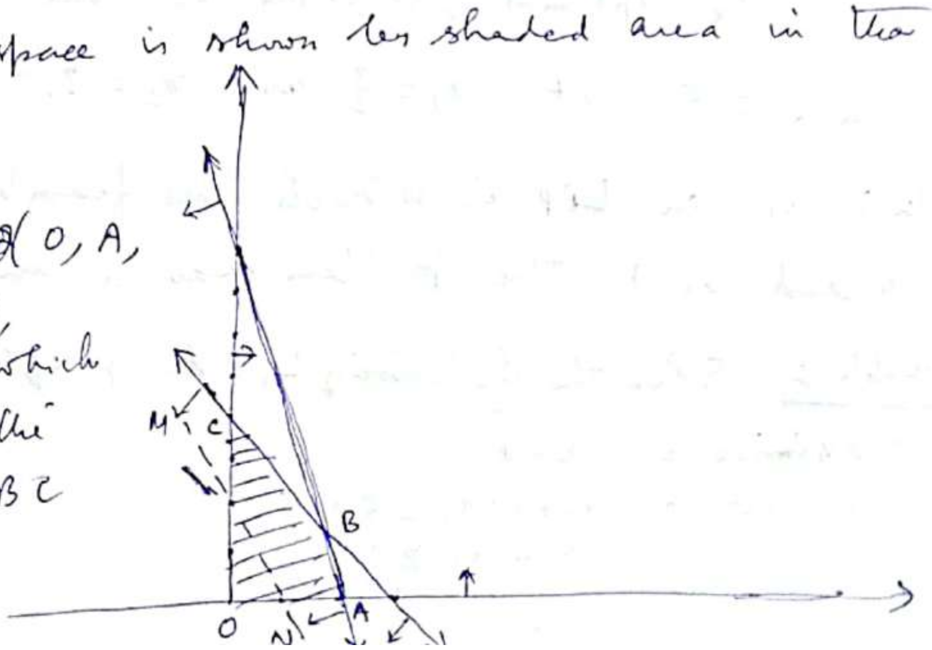
Solution: Changing the inequations to equations we may write

$$4x_1 + 3x_2 = 12 \quad \text{or} \quad \frac{x_1}{3} + \frac{x_2}{4} = 1 \quad \cdots (1)$$

$$4x_1 + x_2 = 8 \quad \text{or,} \quad \frac{x_1}{2} + \frac{x_2}{8} = 1 \quad \cdots (2)$$

The feasible space is shown by shaded area in the adjoining figure

Now the coordinates of O, A, B, C are $(0,0)$, $(2,0)$, $(3/2, 2)$ and $(0,4)$ which are corner points of the shaded region OABC



Now the values of the objective function at these points O, A, B, C are

Z at O is 0

Z at A is 4

Z at B is 5

and Z at C is 4

So the Z_{max} = maximum value of Z = 5 and the optimal solution is $x_1 = \frac{3}{2}$ and $x_2 = 2$

Alternative method : The point $B\left(\frac{3}{2}, 2\right)$ can also be calculated in the following way:

Let the objective function be given any value, say 2. Then the line represented by the objective function is shown by the dotted line MN. Now to find the maximum value of the objective function, this line is moved parallely and it is seen that this line is touching the feasible region at the corner B, the farthest corner of the feasible region from the origin O.

So, the maximum value of Z occurs at B. Now the coordinates of B are $\left(\frac{3}{2}, 2\right)$ and Z at $\left(\frac{3}{2}, 2\right)$ is 5

Thus the optimal solution of the LPP is

$Z_{max} = 5$ at $x_1 = \frac{3}{2}$ and $x_2 = 2$.

This is an LPP in which the feasible region is bounded and the problem has a unique solution.

Example 2 Solve the following LPP by graphical method.

Maximize $Z = x_1 + x_2$

subject to $5x_1 + 10x_2 \le 50$

$x_1 + x_2 \geq 1$

$x_2 \le 4$

$x_1, x_2 \geq 0$